

Use the following to solve questions 1-3 (each 10 points)

Given AR(2) process

$$X_t = \phi_1 X_{t-1} + \phi_2 X_{t-2} + Z_t, \text{ where } Z_t \sim WN(0, \sigma^2).$$

Q.1. Write the Yule-Walker equations for $h=1,2,3$ and find the variance of the process **(10 points)**

Q.2. Find the autocorrelations for lags 1,2,3, $(\rho(1), \rho(2), \rho(3))$ **(5 points)**

Q.3. For the seasonal ARIMA model of order $(1,1,0)*(0,0,2)_2$ write the model in operator form and difference equation form. **(10 points)**

Use following to solve 4-6 (each 10 points)

Consider the model $X_t = \phi X_{t-2} + Z_t - \theta Z_{t-3}$ where $Z_t \sim WN(0, \sigma^2)$.

Q.4 Show if the model is stationary and invertible for $\phi = 4$ and $\theta = 1/8$?

Q.5. Assuming invertibility, find $\pi_1, \pi_2, \pi_3, \pi_4, \pi_5$ in terms of parameters.

Q.6. Find the first three forecasts and the error variances.

Q.7. Explain the following briefly (5 points)

A. Why it is sometimes hard to distinguish between a trend stationary time series and a random walk with intercept when only analyzing the graph?

B. What is the main idea behind cointegration analysis. Discuss also the differences between cointegration and correlation.

Use Outputs I and II to solve questions 8-10 (each 5 points)

Output I

Variable: S_P_500_INDEX

Exogenous: Constant

Lag Length: 0 (Automatic based on SIC, MAXLAG=28)

	t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic	-1.926797	0.3201
Test critical values: 1% level	-3.432187	
5% level	-2.862237	
10% level	-2.567185	

*MacKinnon (1996) one-sided p-values.

Augmented Dickey-Fuller Test Equation

Dependent Variable: D(S_P_500_INDEX)

Method: Least Squares

Date: 06/18/08 Time: 20:01

Sample (adjusted): 1/03/1995 12/31/2007

Included observations: 3228 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
S_P_500_INDEX(-1)	-0.001493	0.000775	-1.926797	0.0541
C	1.942350	0.872543	2.226080	0.0261

R-squared	0.001149	Mean dependent var	0.312655
Adjusted R-squared	0.000840	S.D. dependent var	12.18334
S.E. of regression	12.17822	Akaike info criterion	7.837794
Sum squared resid	478444.8	Schwarz criterion	7.841561
Log likelihood	-12648.20	F-statistic	3.712547
Durbin-Watson stat	2.086200	Prob(F-statistic)	0.054093

Output II

Variable: DIF_S_P_500_INDEX

Exogenous: Constant

Lag Length: 0 (Automatic based on SIC, MAXLAG=28)

	t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic	-59.09627	0.0001
Test critical values: 1% level	-3.432188	
5% level	-2.862237	
10% level	-2.567185	

*MacKinnon (1996) one-sided p-values.

Dependent Variable: D(DIF_S_P_500_INDEX)

Method: Least Squares

Date: 06/18/08 Time: 21:44

Sample (adjusted): 1/04/1995 12/31/2007

Included observations: 3227 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
DIF_S_P_500_INDEX(-1)	-1.039872	0.017596	-59.09627	0.0000
C	0.000375	0.000190	1.973382	0.0485

R-squared	0.519901	Mean dependent var	-2.13E-06
Adjusted R-squared	0.519752	S.D. dependent var	0.015557
S.E. of regression	0.010781	Akaike info criterion	-6.221451
Sum squared resid	0.374839	Schwarz criterion	-6.217683
Log likelihood	10040.31	F-statistic	3492.369
Durbin-Watson stat	2.000633	Prob(F-statistic)	0.000000

Q.8. Write the hypothesis tested in the outputs both in words and notation.(5 points)

H_0 : _____ H_0 : _____

Q.9. Write the decisions in both tests and state the reason for your decision.(5 points)

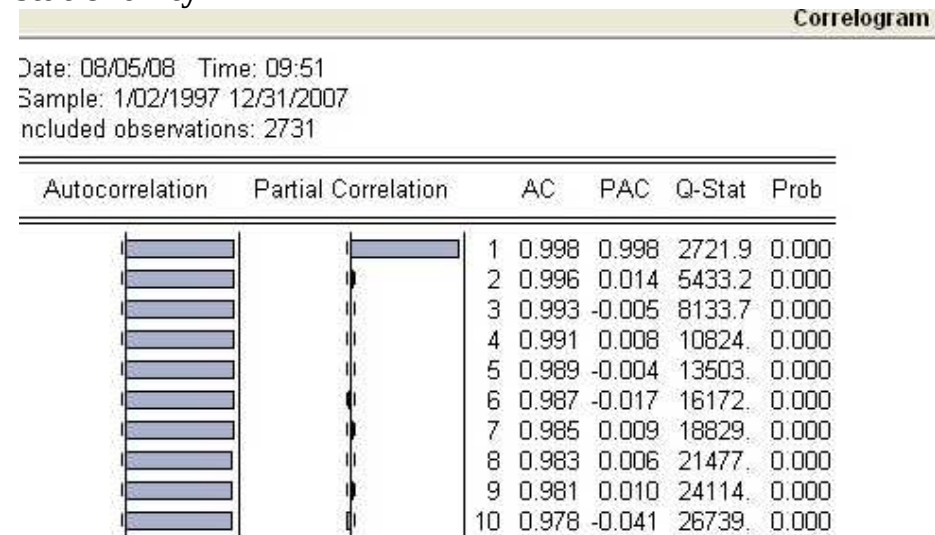
Output 1: _____

Output 2: _____

Q.10. Based on these outputs, can we distinguish if the series is trend stationary or not? Explain how? (5 points)

Use following to solve 11 (10 points)

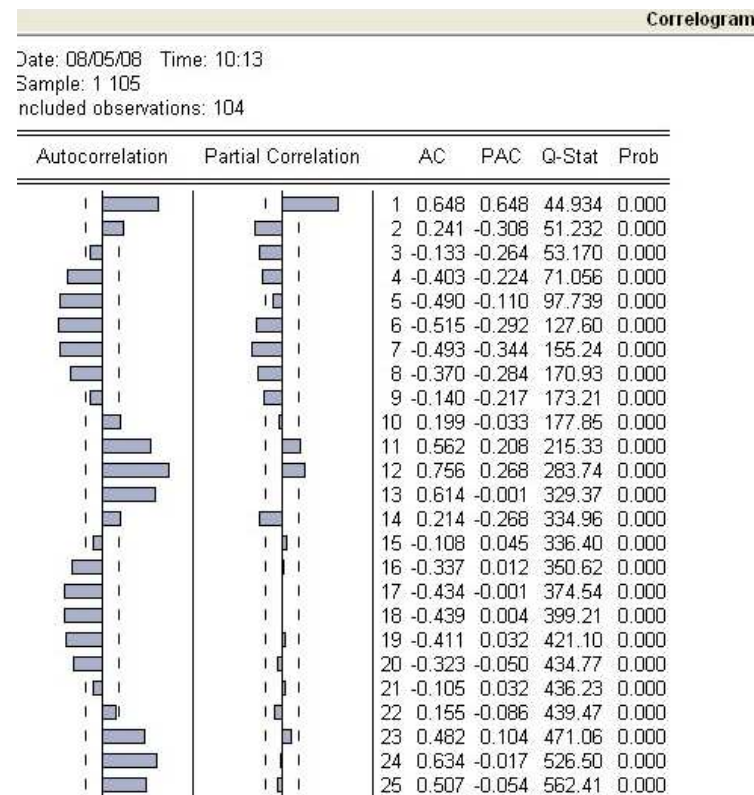
Q.11 For each part given below write the appropriate model, its components (if exists) and comment on their stationarity



A. The model _____

Components: _____

Comment _____



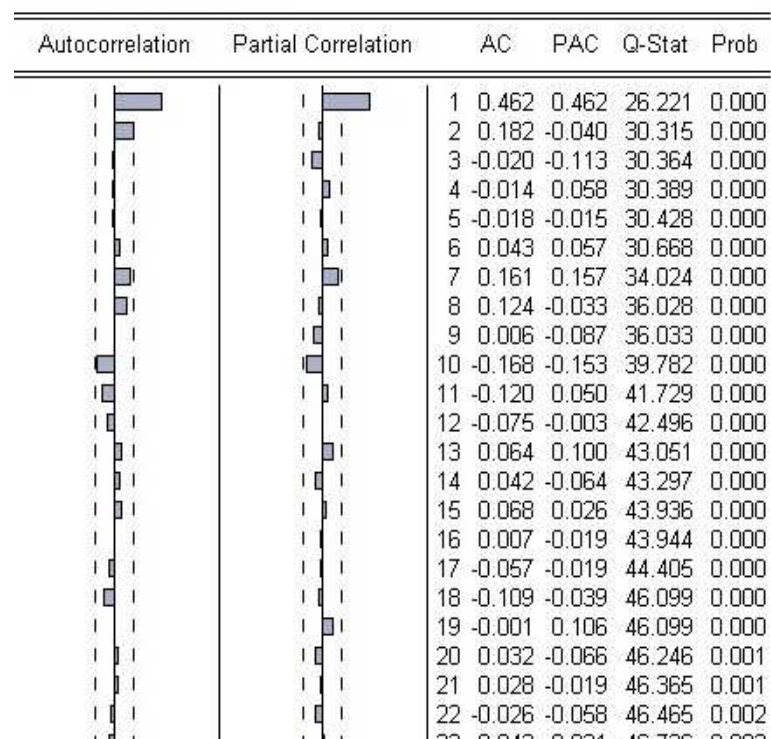
B. The model _____

Component _____

Comment _____

Correlogram

Date: 08/05/08 Time: 10:18
Sample: 1 120
Included observations: 120



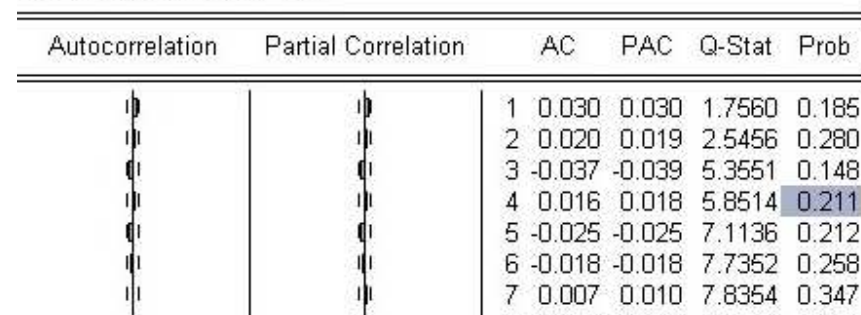
C. The model _____

Components _____

Comments _____

Correlogram

Date: 08/05/08 Time: 09:50
Sample: 1/02/1995 12/31/2007
Included observations: 1994



D. The model is _____

Components _____

Comments _____

Q.12 For each part given below write the model and state which one of the models should be chosen with reasons.

Dependent Variable: DGLOBAL

Method: ML - ARCH (Marquardt) - Normal distribution

Date: 07/22/08 Time: 17:54

Sample (adjusted): 1/06/2000 12/31/2007

Included observations: 1993 after adjustments

Convergence achieved after 26 iterations

Variance backcast: ON

GARCH = C(3) + C(4)*RESID(-1)^2 + C(5)*GARCH(-1)

	Coefficient	Std. Error	z-Statistic	Prob.
C	0.353507	0.222367	1.589745	0.1119
AR(1)	0.061567	0.025442	-2.419941	0.0155
Variance Equation				
C	1.012125	0.238564	4.242566	0.0000
RESID(-1)^2	0.044272	0.006532	6.777346	0.0000
GARCH(-1)	0.949223	0.007228	131.3304	0.0000
R-squared	0.001767	Mean dependent var	0.033241	
Adjusted R-squared	0.000241	S.D. dependent var	13.07855	
S.E. of regression	13.08013	Akaike info criterion	7.741455	
Sum squared resid	340126.7	Schwarz criterion	7.755498	
Log likelihood	7709.360	F-statistic	0.879863	
Durbin-Watson stat	1.981887	Prob(F-statistic)	0.475130	
Inverted AR Roots	-.06			

MODEL 1

Dependent Variable: DGLOBAL

Method: ML - ARCH (Marquardt) - Normal distribution

Date: 07/22/08 Time: 17:55

Sample (adjusted): 1/13/2000 12/31/2007

Included observations: 1989 after adjustments

Convergence achieved after 21 iterations

Variance backcast: ON

GARCH = C(4) + C(5)*RESID(-1)^2 + C(6)*GARCH(-1)

	Coefficient	Std. Error	z-Statistic	Prob.
C	0.346259	0.214226	1.616330	0.1060
AR(1)	-0.062668	0.025534	-2.454306	0.0141
AR(5)	-0.047644	0.023200	-2.053601	0.0400
Variance Equation				
C	1.064236	0.253974	4.190330	0.0000
RESID(-1)^2	0.046589	0.006826	6.824785	0.0000
GARCH(-1)	0.946787	0.007575	124.9805	0.0000
R-squared	0.003643	Mean dependent var	0.018155	
Adjusted R-squared	0.001131	S.D. dependent var	13.06297	
S.E. of regression	13.05558	Akaike info criterion	7.739272	
Sum squared resid	337998.8	Schwarz criterion	7.756151	
Log likelihood	-7690.706	F-statistic	1.450252	
Durbin-Watson stat	1.981132	Prob(F-statistic)	0.203151	
Inverted AR Roots	.43+.32i	.43-.32i	-.18-.52i	-.18+.52i

MODEL 2

Q.12 For each part given below write the model and state which one of the models should be chosen with reasons (10 points).

MODEL 1: _____

MODEL 2: _____

MODEL CHOSEN: _____

REASONS: _____

Q.13. Interpret the Q-statistics and state the relevant hypothesis for the following Correlogram. (5 points)

Correlogram of Standardized Residuals Squared

Date: 07/22/08 Time: 18:01

Sample: 1/13/2000 12/31/2007

Included observations: 1989

Q-statistic probabilities adjusted for 2 ARMA term(s)

Autocorrelation	Partial Correlation	AC	PAC	Q-Stat	Prob
		1 -0.037	-0.037	2.7648	
		2 0.012	0.011	3.0487	
		3 0.002	0.003	3.0596	0.080
		4 0.026	0.026	4.4236	0.110
		5 0.018	0.019	5.0381	0.169
		6 -0.012	-0.011	5.3059	0.257
		7 -0.005	-0.006	5.3561	0.374
		8 0.015	0.014	5.8236	0.443
		9 -0.008	-0.008	5.9611	0.544
		10 0.053	0.053	11.628	0.169
		11 -0.003	0.001	11.651	0.234
		12 0.013	0.011	11.999	0.285
		13 -0.014	-0.013	12.366	0.337
		14 -0.003	-0.007	12.389	0.415
		15 -0.030	-0.033	14.241	0.357
		16 0.023	0.022	15.339	0.355
		17 -0.010	-0.007	15.550	0.413
		18 0.012	0.011	15.839	0.464
		19 0.011	0.014	16.083	0.518
		20 -0.010	-0.013	16.283	0.573
		21 0.006	0.004	16.347	0.634
		22 -0.019	-0.020	17.096	0.647
		23 0.006	0.006	17.163	0.701
		24 0.006	0.007	17.245	0.750
		25 -0.024	-0.019	18.426	0.734

Answer: