TIME SERIES ANALYSIS - FORMULA SHEET

$S_t = wY_t + (1-w)S_{t-1}$; $MAD = \frac{1}{n}\sum_{t=1}^{n}\left|y_t - \hat{y}_t\right|$; $SSE = \frac{1}{n}\sum_{t=1}^{n}\left(y_t - \hat{y}_t\right)^2$; $JB = \left(\frac{T}{6}\right)\left(\hat{s}^2 + \frac{1}{4}(\hat{k}-3)^2\right)$

$\gamma(r,s) = Cov(X_r, X_s) = \gamma_x(r,s) = E[(X_r - \mu_x(r))(X_s - \mu_x(s))]$; $Cov(X_r, X_r) = \gamma(0) = Var[X_t]$

$Q^*(m) = T(T+2)\sum_{j=1}^{m}\frac{\hat{\rho}_j^2}{T-j}$; $Q(m) = T\sum_{j=1}^{m}\hat{\rho}_j^2$; $\rho_h = \frac{\gamma(h)}{\gamma(0)} = \frac{Cov(X_{t+h}, X_t)}{Cov(X_t, X_t)}$; $X_t = \mu + \sum_{j=-\infty}^{\infty}\psi_j Z_{t-j}$

$X_t = \varphi_1 X_{t-1} + \varphi_2 X_{t-2} + .. + \varphi_p X_{t-p} + Z_t + \theta_1 Z_{t-1} + \theta_2 Z_{t-2} + .. + Z_q X_{t-q}$

$$E[Z_t] = 0 \quad \gamma(h) = \begin{cases} \sigma^2(1+\theta^2) & h = 0 \\ \sigma^2\theta & h = \mp 1 \\ 0 & |h| > 1 \end{cases} \qquad \rho_x(h) = \begin{cases} 1 & h = 0 \\ \frac{\theta}{1+\theta^2} & h = \mp 1 \\ 0 & |h| > 1 \end{cases}$$

$$\phi_{hh} = \frac{\theta^h(1-\theta^2)}{1-\theta^{2(h+1)}} \quad for \quad h > 0$$

$$\phi_{11} = \frac{\theta(1-\theta^2)}{1-\theta^4} \quad \phi_{22} = \frac{\theta^2(1-\theta^2)}{1-\theta^6} \quad \phi_{33} = \frac{\theta^3(1-\theta^2)}{1-\theta^8}$$

$$E[X_t] = 0 \qquad \gamma(h) = \left\{ \gamma(h) = \frac{\sigma^2\varphi^h}{1-\varphi^2} \right. \quad h > 0 \quad \rho(h) = \varphi^h \;\; AIC = \ln\hat{\sigma}_k^2 + \frac{n+2k}{n}$$

$$\phi_{hh} = \frac{\rho(h) - \sum_{j=1}^{h-1}\phi_{h-1,j}\;\rho_{h-j}}{1 - \sum_{j=1}^{h-1}\phi_{h-1,j}\;\rho_j} \quad h = 1,2,3,\cdots; \quad SIC = \ln\hat{\sigma}_k^2 + \frac{k\ln n}{n} \;; \quad DW = \frac{\sum_{t=2}^{n}(\varepsilon_t - \varepsilon_{t-1})^2}{\sum_{t=1}^{n}\varepsilon_t^2}$$

$$(1-\varphi_1 B - .. - \varphi_p B^p)(1-B)^d(1-\lambda B^s)^D X_t = (1+\theta_1 B + .. + \theta_q B^q)(1+\omega B^s)^D Z_t$$

$$X_n(k) = E[X_{n+k} \mid F_n] = \hat{X}_n(k) = E[X_{n+k} \left| F_n\right.] = \sum_{i=1}^{p}\psi_i \hat{X}_n(k-i) + \sum_{j=0}^{q}\theta_j Z_n(k-i)$$

$$e_n(l) = X_{n+l} - \hat{X}_n(l) = Z_{n+l} + \psi_1 Z_{n+l-1} + .. + \psi_{l-1}Z_{n+1}; \quad Var[e_n(l)] = \sigma^2(1+\psi_1^2 + \psi_2^2 + .. + \psi_{l-1}^2)$$

$$E[Z_{n+j} \mid X_n, X_{n-1}, \cdots] = \begin{cases} Z_{n+j} & j \le 0 \\ 0 & j > 0 \end{cases} \begin{bmatrix} y_{1t} \\ y_{2t} \end{bmatrix} = \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} + \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}\begin{bmatrix} y_{1,t-1} \\ y_{2,t-1} \end{bmatrix} + \begin{bmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \end{bmatrix};$$

$$K = \beta_\infty + \beta_1 T^{-1} + \beta_2 T^{-2}$$

$$X_{1,2} = \frac{b \pm \sqrt{b^2 - 4ac}}{-2a} \;; \; \sum_{i=0}^{\infty} r^i = \frac{1}{1-r}, \; |r| < 1 \;; \quad \begin{aligned} \Delta y_{1t} &= \lambda_1\varepsilon_{t-1} + \sum_{i=1}^{p-1}(a_{11i}\Delta y_{1t-i} + a_{12i}\Delta y_{2t-i}) + \varepsilon_{1t} \\ \Delta y_{2t} &= \lambda_2\varepsilon_{t-1} + \sum_{i=1}^{p-1}(a_{21i}\Delta y_{1t-i} + a_{22i}\Delta y_{2t-i}) + \varepsilon_{1t} \end{aligned}$$