

Financial Data Analysis

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Financial Data Analysis

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Office hour: Tuesday 10-12, Room: 2303, Platz der Alten Synagoge
- Exercise course: Every other week (on average!)
- In the exercise course, participants present solutions to either theoretical or small empirical problems.
- By doing so, you can also earn a bonus on the final course grade.
- There will be 80 points altogether in the final exam.
- You can earn

$$p^{\star} = 6 \times k^{2/3} \tag{1}$$

bonus points, where k is the number of times presented.

Prerequisites: “Einführung in die Empirische Wirtschaftsforschung”, “Ökonometrie 1” or “Applied Econometrics”

Course Outline

- Introduction: Basic properties of financial return series
- Time series basics
- Parametric volatility modeling
 - (i) GARCH
 - (ii) Stochastic volatility models
 - (iii) Regime-switching models
- Modeling the dependence structure of returns
 - (i) Multivariate GARCH processes
 - (ii) Multivariate regime-switching and copulas
- Further topics, e.g., Value-at-Risk (Regulatory framework, quantile estimation, backtesting)

Textbooks and Articles

- Taylor, S. (2005), Asset Price Dynamics, Volatility, and Prediction, Princeton University Press, New Jersey.
- Alexander, C. (2008), Practical Financial Econometrics, Wiley, New York
- Tsay, R.S. (2005), Analysis of Financial Time Series, 2e, Wiley, New York.
- Hamilton (1994), Time Series Analysis, Princeton University Press, Princeton, New Jersey.
- Mills, T. C., Markellos (2008), The Econometric Modelling of Financial Time Series, 2e, Cambridge University Press.
- Campbell, Lo, and MacKinlay (1997), The Econometrics of Financial Markets, Princeton University Press.
- Franses, van Dijk (2000), Non-Linear Time Series Models in Empirical Finance, Cambridge University Press.

- Timo Teräsvirta (2009): An introduction to univariate GARCH models, Handbook of Financial Time Series, Springer, downloadable at <http://ideas.repec.org/p/hhs/hastef/0646.html>.
- Annastiina Silvennoinen and Timo Teräsvirta (2009): Multivariate GARCH Models, Handbook of Financial Time Series, Springer, downloadable at <http://ideas.repec.org/p/hhs/hastef/0669.html>.
- Malevergne, Sornette (2006), Extreme Financial Risks, From Dependence to Risk Management, Springer, Berlin. (For copulas)

Returns

- Let P_t be the asset price at time t (stock, stock index, exchange rate,...).
- There is a dividend payment D_t at the end of period t .
- Then the (single-period) *discrete return* is

$$R_t = \frac{P_t + D_t - P_{t-1}}{P_{t-1}}. \quad (2)$$

- Dividends are often excluded from return calculations (price index vs. total return index).
- Often (2) is multiplied by 100 to be interpretable in terms of *percentage returns*.

- The *continuously compounded* or *log* return is (ignoring dividends for simplicity)

$$r_t = \log P_t - \log P_{t-1} = \log(1 + R_t). \quad (3)$$

- This name derives from the fact that the interest rate i_n equivalent to R_t , when interest is paid n times in the period, solves

$$\left(1 + \frac{i_n}{n}\right)^n = 1 + R_t. \quad (4)$$

- Continuous compounding is approached as $n \rightarrow \infty$, and then

$$e^{i_\infty} = 1 + R_t \Rightarrow i_\infty = \log(1 + R_t) = r_t. \quad (5)$$

- Recall that

$$e^x = \lim_{n \rightarrow \infty} \left(1 + \frac{x}{n}\right)^n = \sum_{i=0}^{\infty} \frac{x^i}{i!}. \quad (6)$$

- Empirical analysis is often based on log returns. These have the advantage that they can be additively aggregated over time.
- That is, if $r_{t,t+\tau}$ denotes the (multi-period) return from time t to time $t + \tau$, we have

$$r_{t,t+\tau} = \log \left(\frac{P_{t+\tau}}{P_t} \right) = \log \left(\frac{P_{t+\tau}}{P_{t+\tau-1}} \frac{P_{t+\tau-1}}{P_{t+\tau-2}} \dots \frac{P_{t+1}}{P_t} \right) \quad (7)$$

$$= \sum_{i=1}^{\tau} \log \left(\frac{P_{t+i}}{P_{t+i-1}} \right) = \sum_{i=1}^{\tau} r_{t+i}. \quad (8)$$

- This is not the case for the discrete return, where

$$R_{t,t+\tau} = \prod_{i=1}^{\tau} (1 + R_{t+i}) - 1. \quad (9)$$

- On the other hand, if we consider a portfolio of N assets with weights a_i , and returns R_{it} , $i = 1, \dots, n$, then the portfolio return is

$$R_{p,t} = \sum_{i=1}^N a_i R_{it}, \quad (10)$$

whereas

$$r_{p,t} = \log(1 + R_{p,t}) \neq \sum_{i=1}^N a_i r_{it}, \quad (11)$$

i.e., the linear combination of continuously compounded asset returns is not the continuously compounded portfolio return.

- For small x ,¹

$$\log(1 + x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots \approx x, \quad (12)$$

so that r_t may serve as a reasonable approximation to the discrete return.

¹Note that the expansion (not the approximation) is only valid for $x \in (-1, 1]$.

Table 1: Discrete and continuous returns

$100 \times R_t$	-30.0	-20.0	-15.0	-10.0	-5.0	0	5.0	10.0	15.0	20.0	30.0
$100 \times r_t$	-35.7	-22.3	-16.3	-10.5	-5.1	0	4.9	9.5	14.0	18.2	26.2

R_t and $r_t = \log(1 + R_t)$ are the discrete and continuously compounded returns, respectively.

- The approximation

$$r_{p,t} \approx \sum_{i=1}^N a_i r_{it} \quad (13)$$

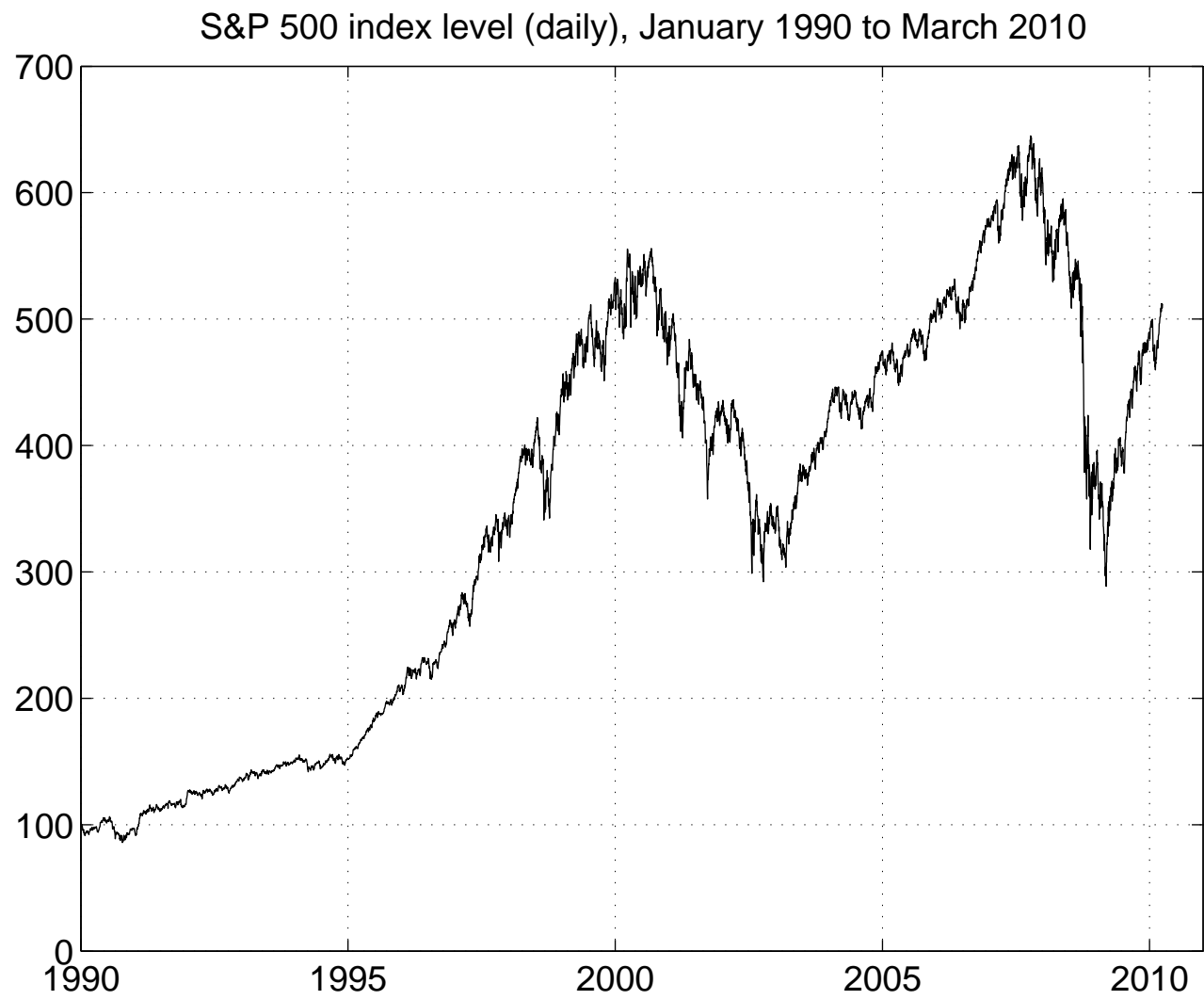
is also frequently used.

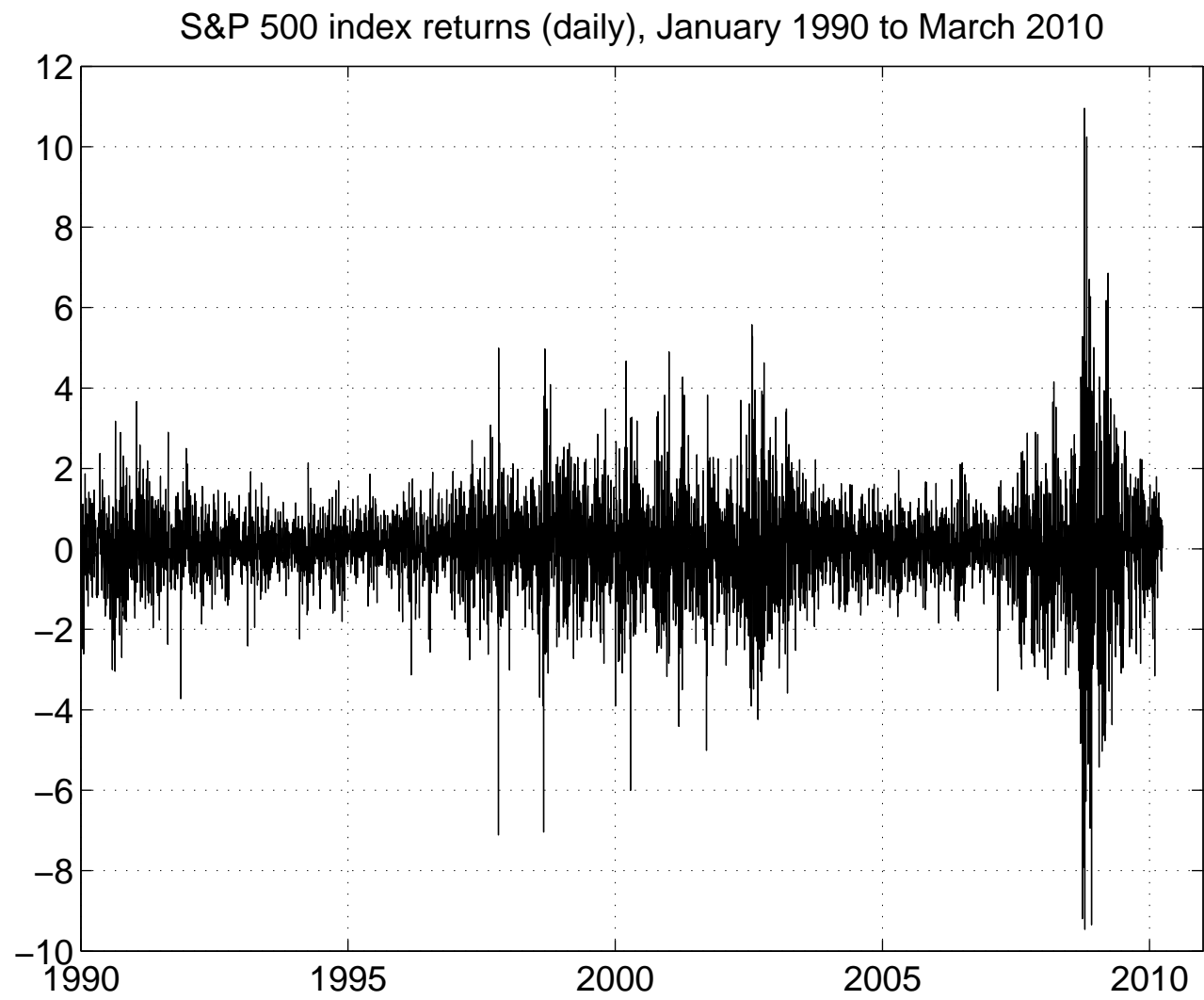
Basic Statistical Properties of Returns: Return Distribution

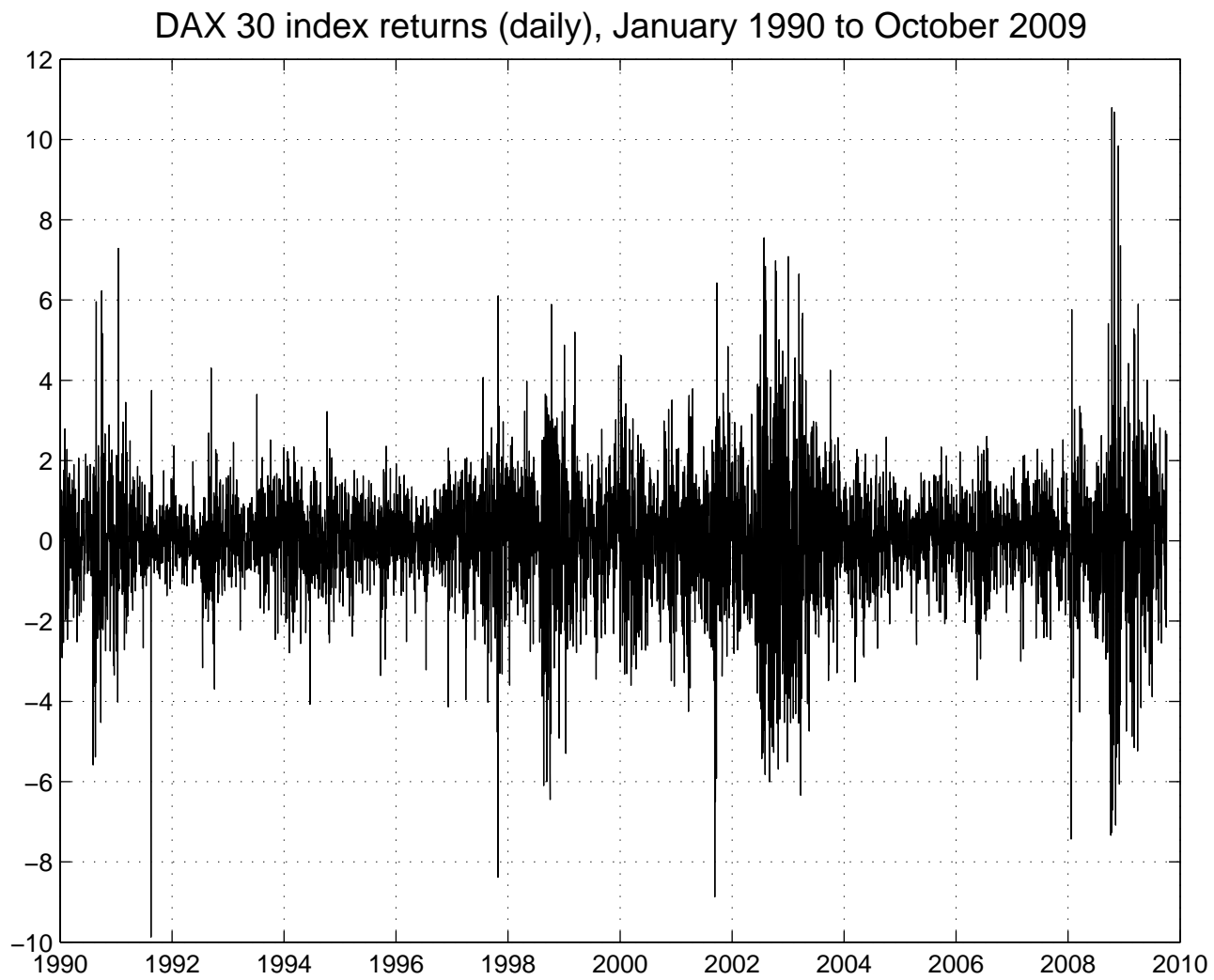
- The traditional assumption that has long dominated empirical finance was that log-returns over longer time intervals are approximately normally distributed.
- For example, daily returns are the sum of a large number of intraday returns.
- Appealing to the central limit theorem, Osborne (1959) argued in a classical article that²

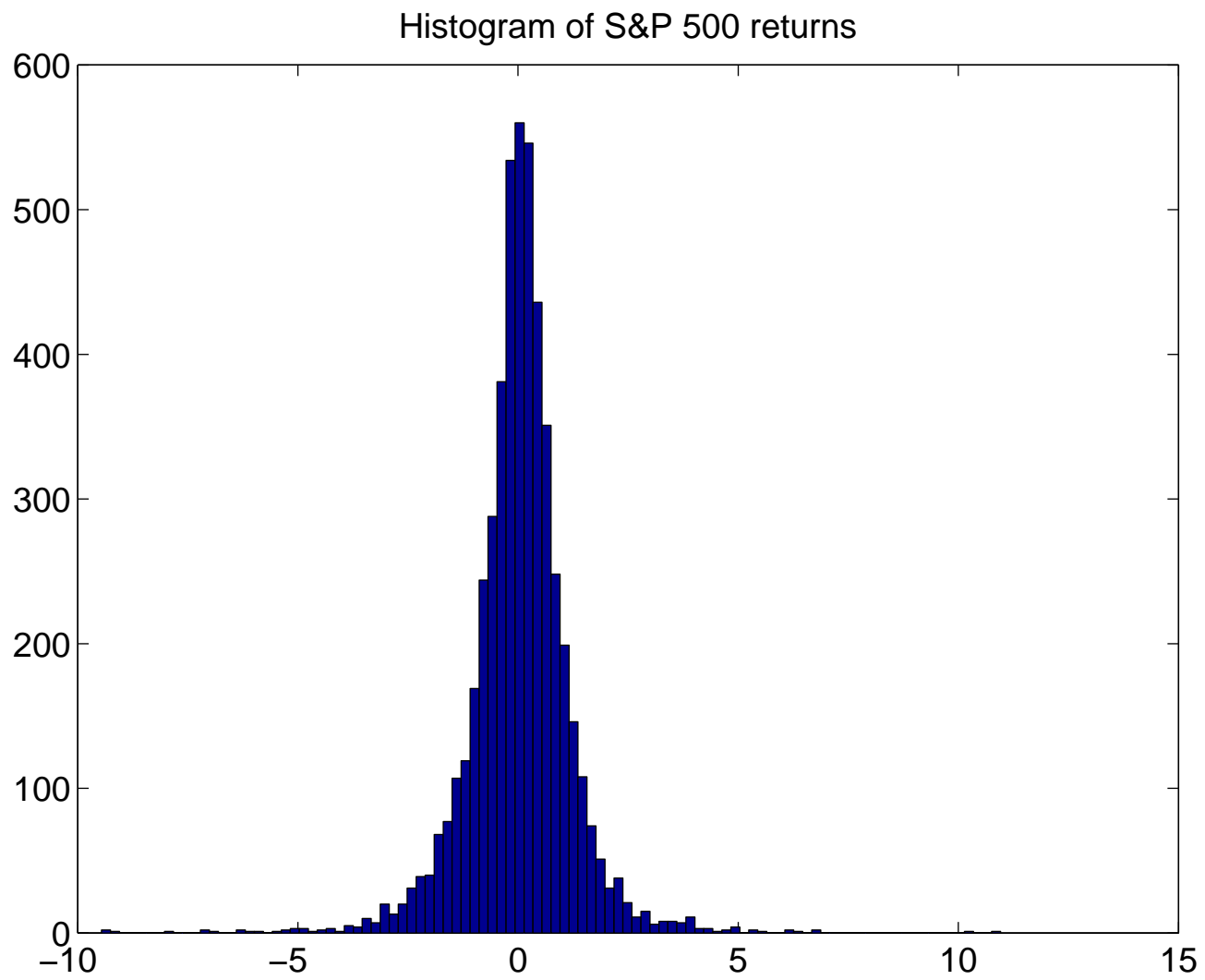
“under fairly general conditions [...] we can expect that the distribution function of $[r_t]$ will be normal.”

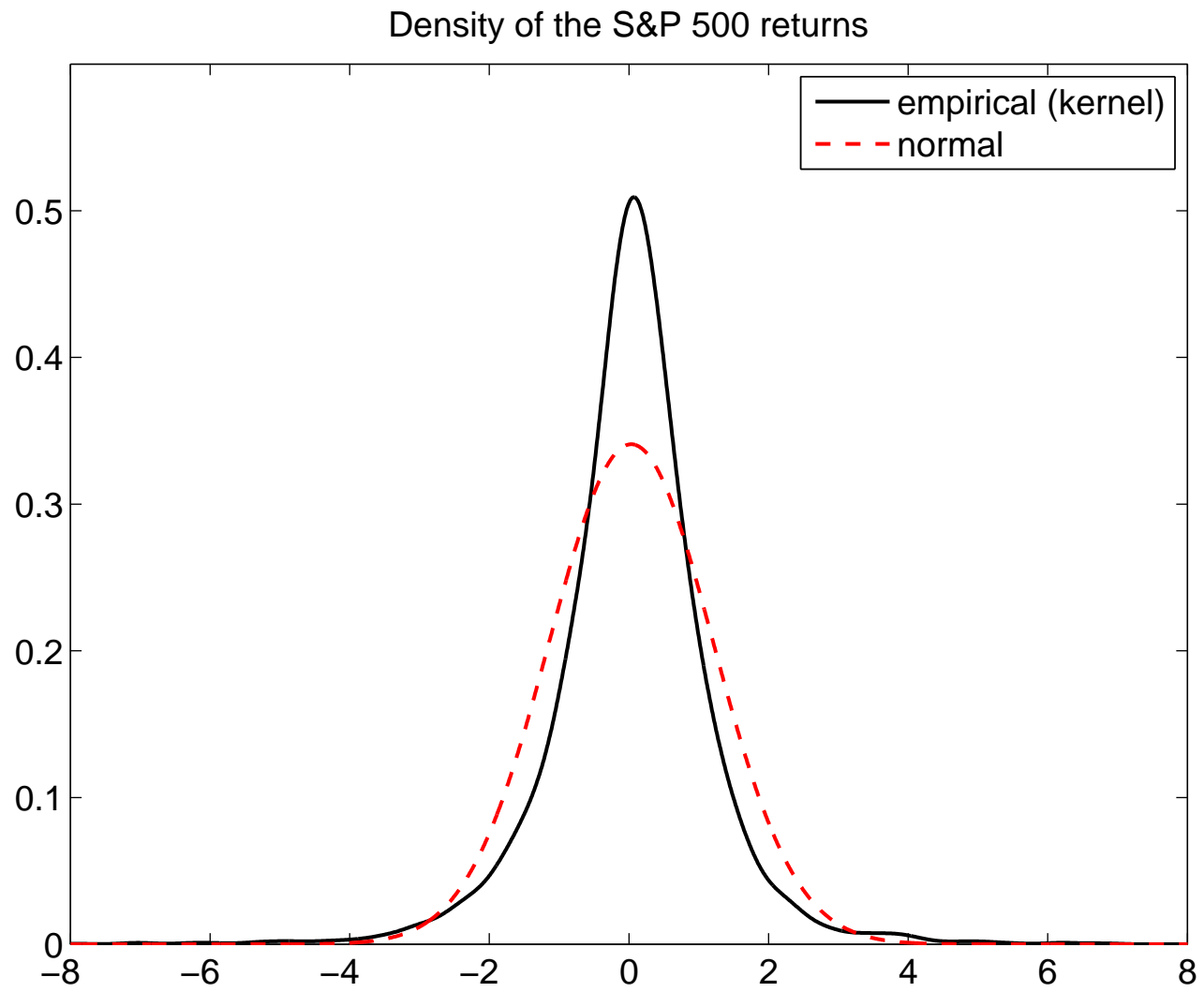
²Brownian Motion in the Stock Market, *Operations Research* 7, 145-173.

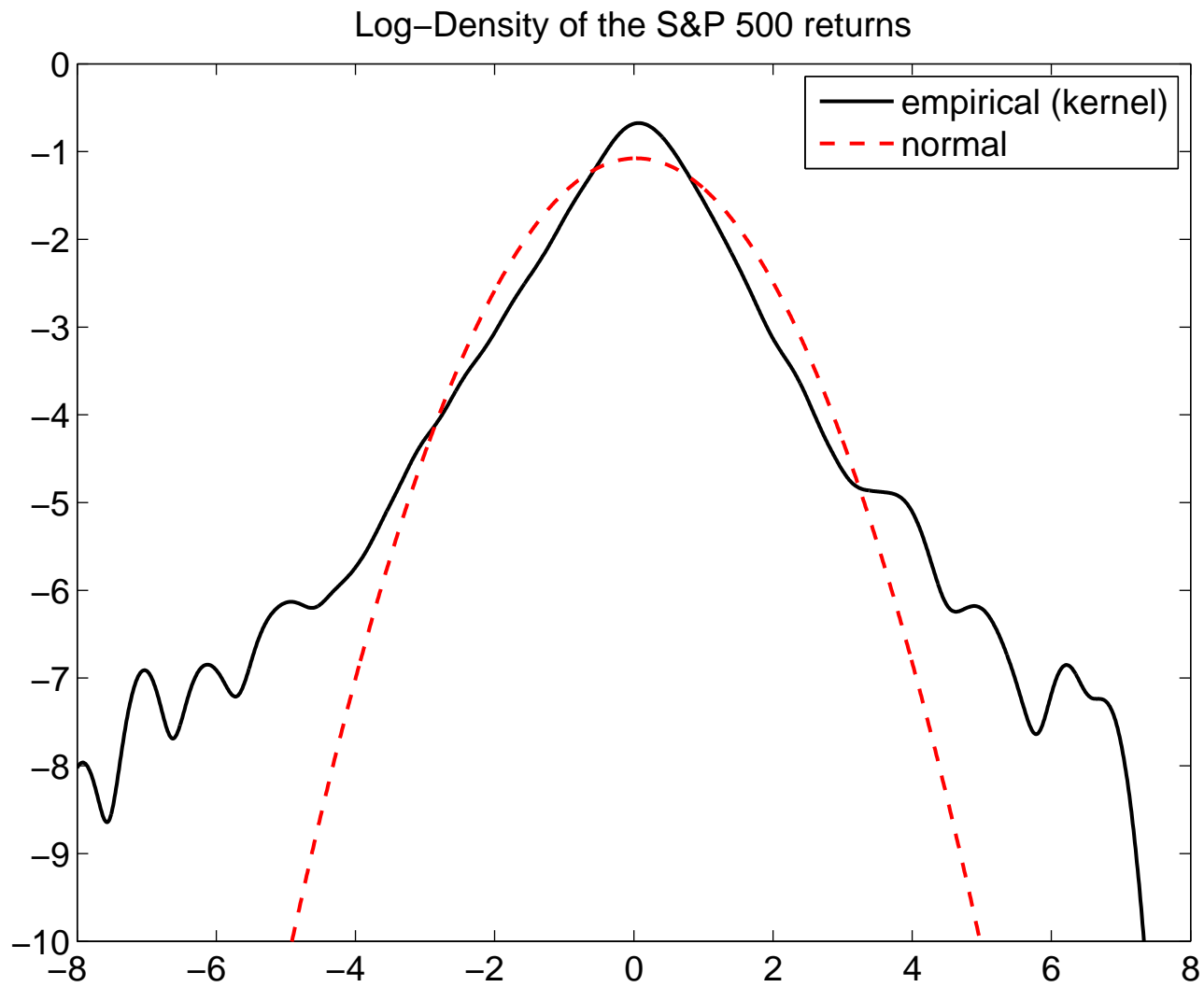


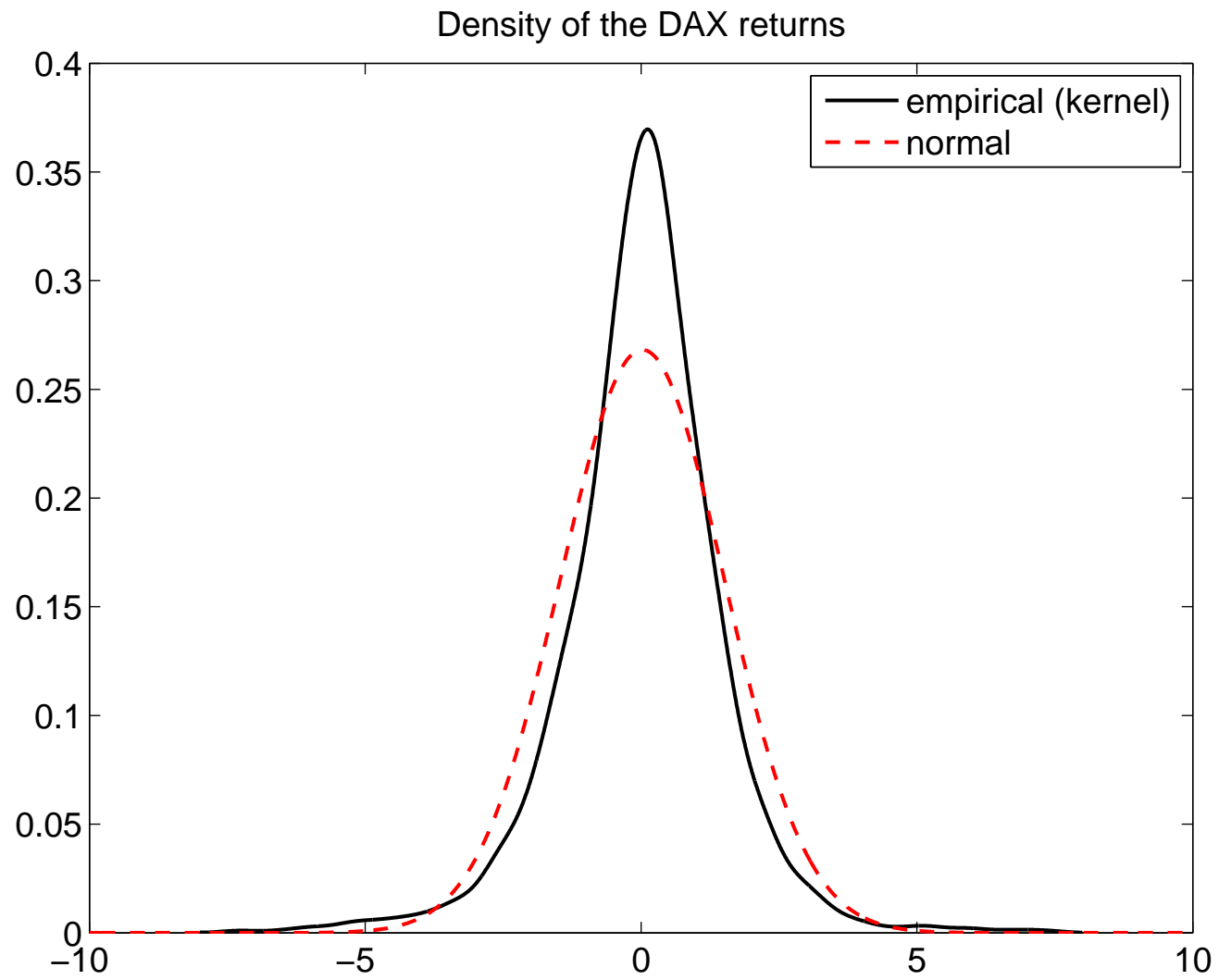


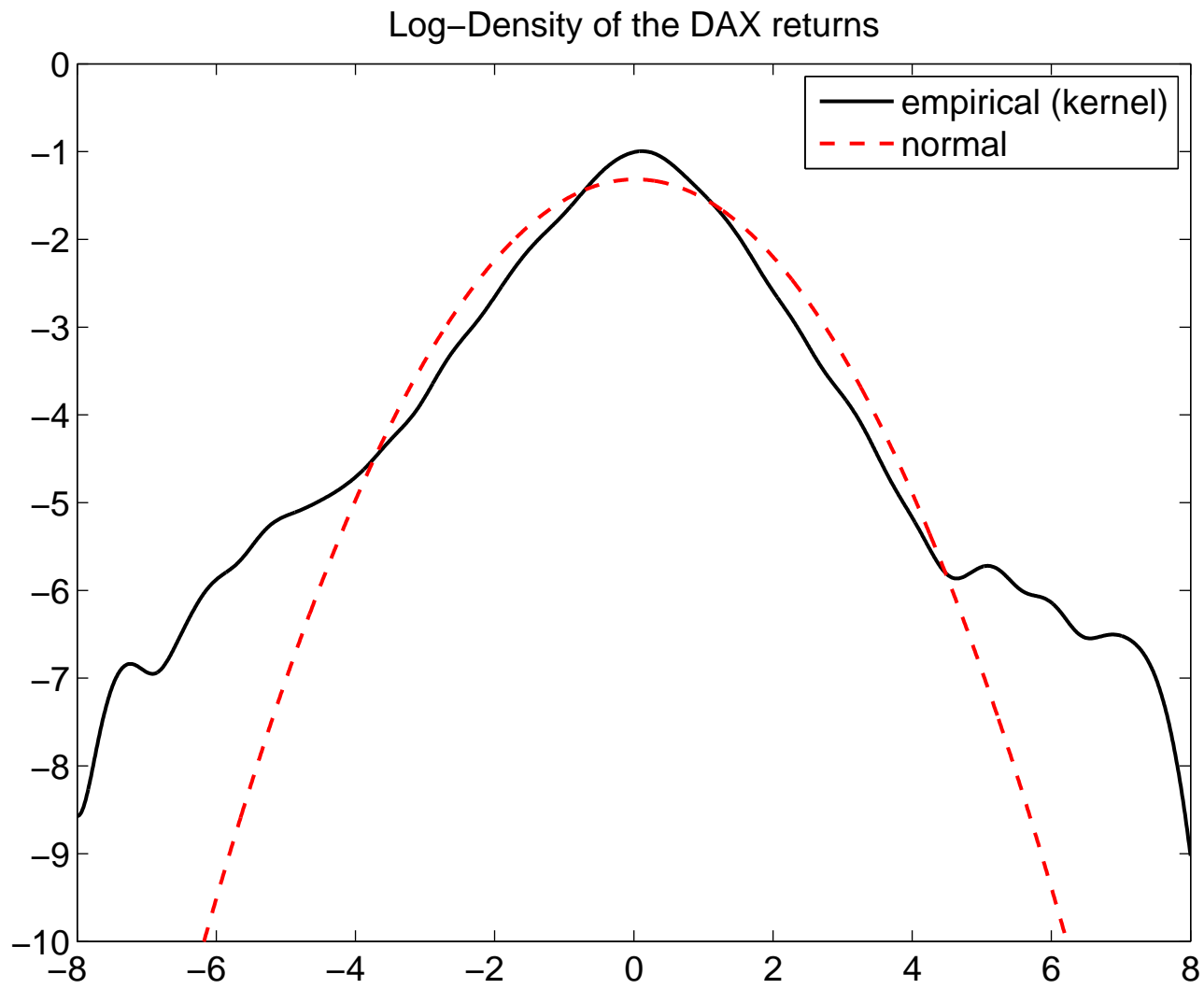








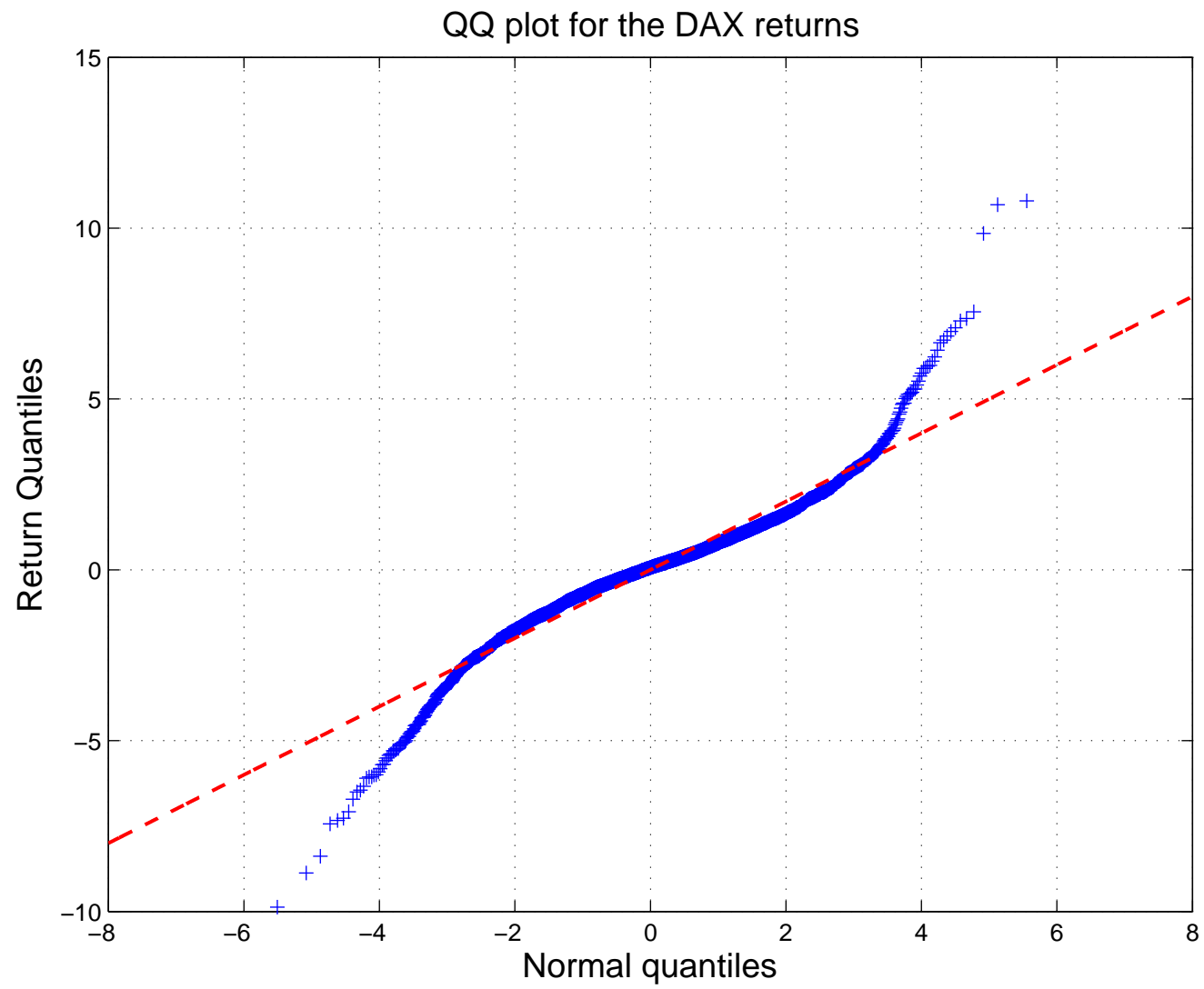


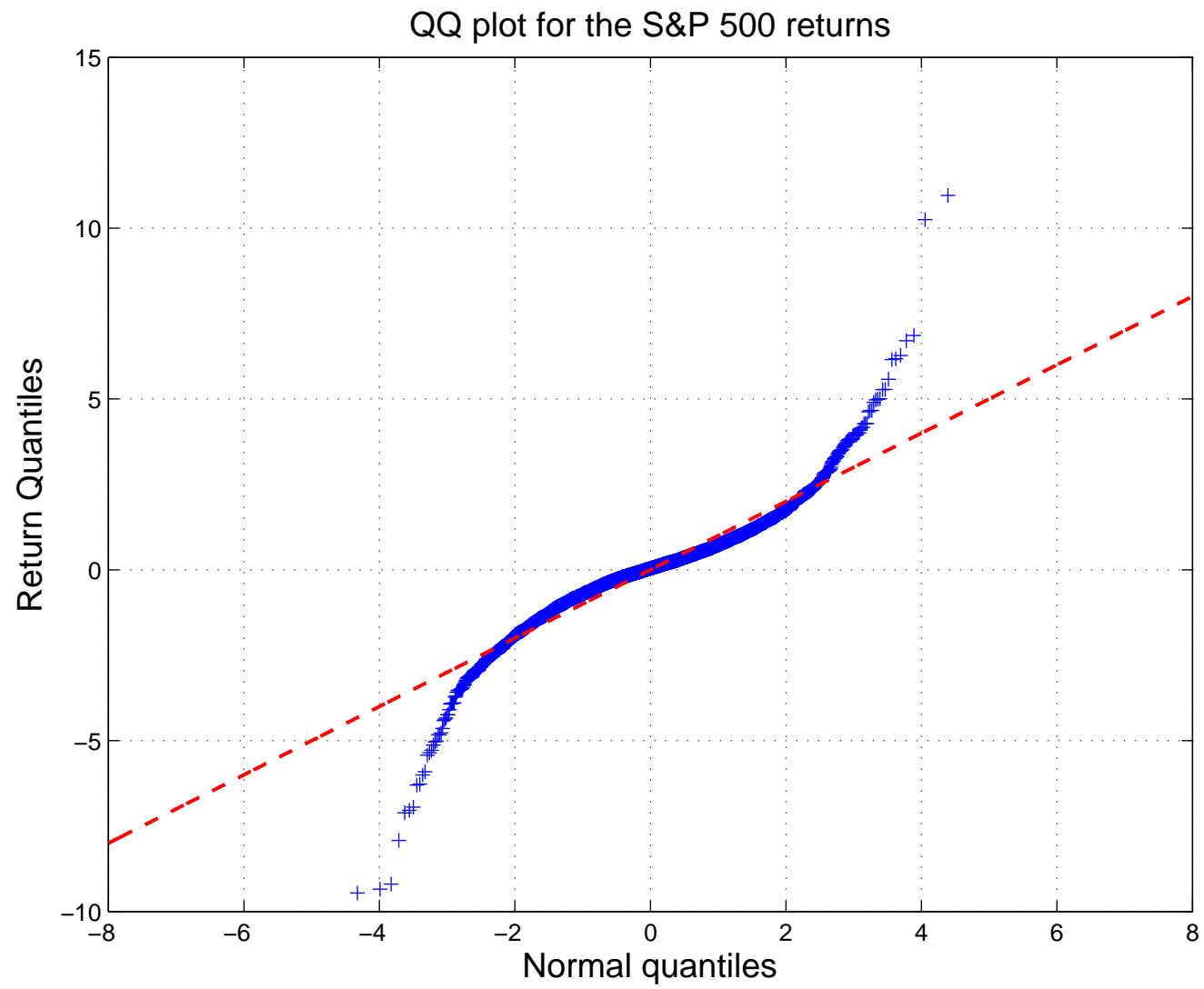


Basic Statistical Properties of Returns: Return Distribution

- Financial Returns at higher frequencies (higher than a month at least) are not normally distributed.
- In particular, they have much more probability mass in the center and the tails than a normal distribution with the same variance.
- This implies, among other things, that the probability of large losses is much higher than under the Gaussian assumption.
- At lower frequencies, however, the central limit theorem appears to operate, and the return distribution begins to closer resemble a Gaussian shape.

- A further simple device for detecting departures from normality (or any other hypothesized distribution) are QQ plots.
- This is a scatter plot of the empirical quantiles (vertical axis) against the theoretical quantiles (horizontal axis) of a given distribution (e.g., the normal distribution).
- Excess kurtosis means that the probability of large negative or positive values is greater than under the corresponding normal density function. So the lower quantiles are smaller than the normal quantiles, and the upper quantiles are greater.
- Consequently, fat tails show up in QQ plots as deviations below an ideal straight line at the lower quantiles, and above the straight line at the upper quantiles.





Kurtosis

- A distribution with higher peaks and fatter tails (and, consequently, less mass in the shoulders) than the normal is called “leptokurtic”.
- The standardized fourth moment is often calculated to measure the degree of leptokurtosis, i.e.,

$$\kappa = \text{kurtosis}(r) = \frac{\mathbb{E}(r - \mu)^4}{\sigma^4}, \quad (14)$$

where μ and σ^2 are the mean and variance of r , respectively, and the sample analogue is

$$\hat{\kappa} = \frac{T^{-1} \sum_{t=1}^T (r_t - \bar{r})^4}{\hat{\sigma}^4}, \quad (15)$$

where \bar{r} is the sample mean.

- For the normal distribution, $\kappa = 3$, and a distribution with $\kappa > 3$ is then classified as leptokurtic.
- The intuition is that the contribution of the rare and large returns in the tails is larger for the fourth moment than for the second (variance).

Skewness

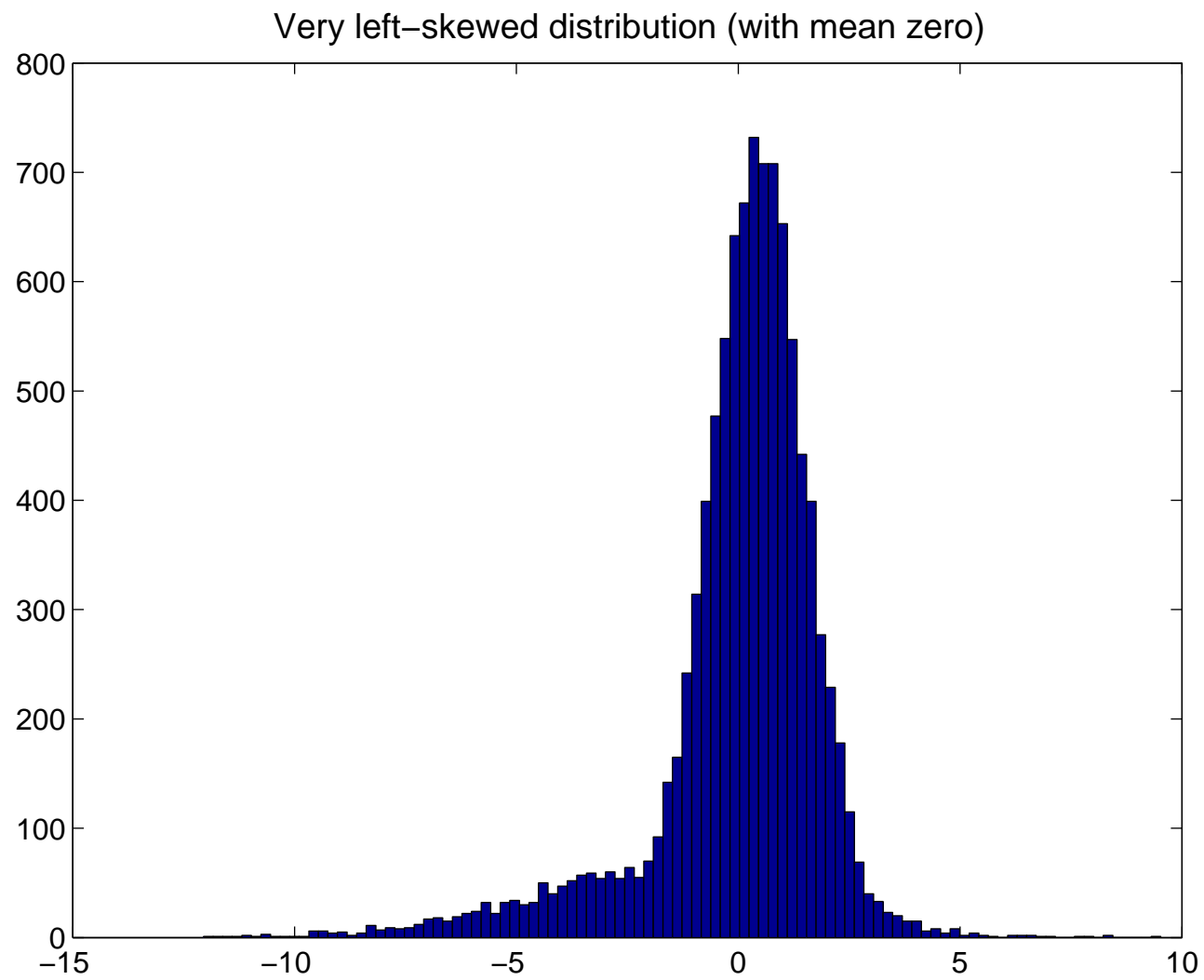
- Sometimes we also observe deviations from symmetry, although these tend to be less pronounced and more difficult to detect.
- The moment-based skewness measure is

$$s = \text{skewness}(r) = \frac{\mathbb{E}(r - \mu)^3}{\sigma^3}, \quad (16)$$

with sample counterpart

$$\hat{s} = \frac{T^{-1} \sum_{t=1}^T (r_t - \mu)^3}{\hat{\sigma}^3}. \quad (17)$$

- For the normal (and any other symmetric density), $s = 0$.
- For example, if $s < 0$, then negative tail observations dominate, and the distribution is skewed to the left.



Jarque–Bera test for normality

- Measures $\hat{\kappa}$ and \hat{s} can be used to construct a test for normality.
- Under normality, $\hat{s} \overset{asy}{\sim} \text{Normal}(0, 6/T)$, and $\hat{\kappa} \overset{asy}{\sim} \text{Normal}(3, 24/T)$, so

$$T\hat{s}^2/6 \overset{asy}{\sim} \chi^2(1), \quad T(\hat{\kappa} - 3)^2/24 \overset{asy}{\sim} \chi^2(1), \quad (18)$$

and both are asymptotically independent, so

$$JB = T\hat{s}^2/6 + T(\hat{\kappa} - 3)^2/24 \overset{asy}{\sim} \chi^2(2), \quad (19)$$

a χ^2 distribution with two degrees of freedom.

- Note that we cannot easily use \hat{s} as a basis for a test of symmetry. Although symmetric distributions always have $s = 0$, the asymptotic standard error $\sqrt{6/T}$ is valid only under normality, and it is much larger for fat-tailed symmetric distributions.

Alternative Distributions for Returns

- Mandelbrot (1963),³ in a famous study of cotton price changes, was one of the first to point out the fat-tailed nature of return distributions.
- Mandelbrot suggested (nonnormal) α -stable (or *stable Paretian*) distributions as a generic model for asset returns, which may be viewed as a generalization of Osborne's Gaussian model.

³B. Mandelbrot (1963). The Variation of Certain Speculative Prices. *Journal of Business* 36, 394-419.

Alternative Distributions for Returns: Discrete Normal Mixtures

- A k -component (discrete) normal mixture distribution is described by the density

$$f(x) = \sum_{j=1}^k \lambda_j \phi(x; \mu_j, \sigma_j^2), \quad \phi(x; \mu, \sigma^2) = \frac{1}{\sqrt{2\pi}\sigma} \exp \left\{ -\frac{(x - \mu)^2}{2\sigma^2} \right\}, \quad (20)$$

$\lambda_j > 0$, $j = 1, \dots, k$, are the *mixing weights*, satisfying $\sum_j \lambda_j = 1$, and the μ_j s and σ_j^2 s are the component means and variances respectively.

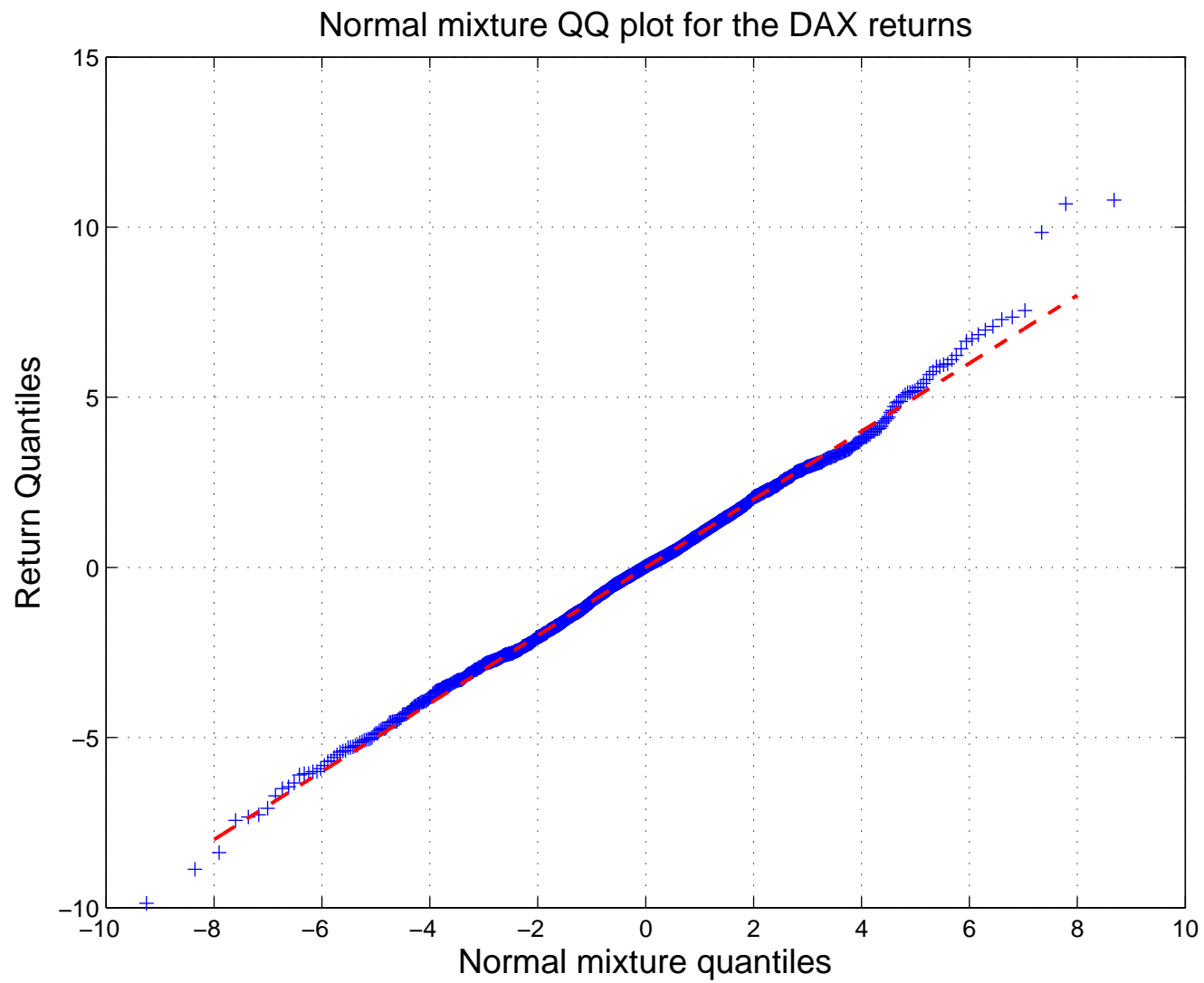
- Flexible with respect to skewness and kurtosis.
- A possible interpretation of the normal mixture is that returns are normally distributed, but that return expectation and variance depend on the market regime, e.g., bull vs. bear markets.

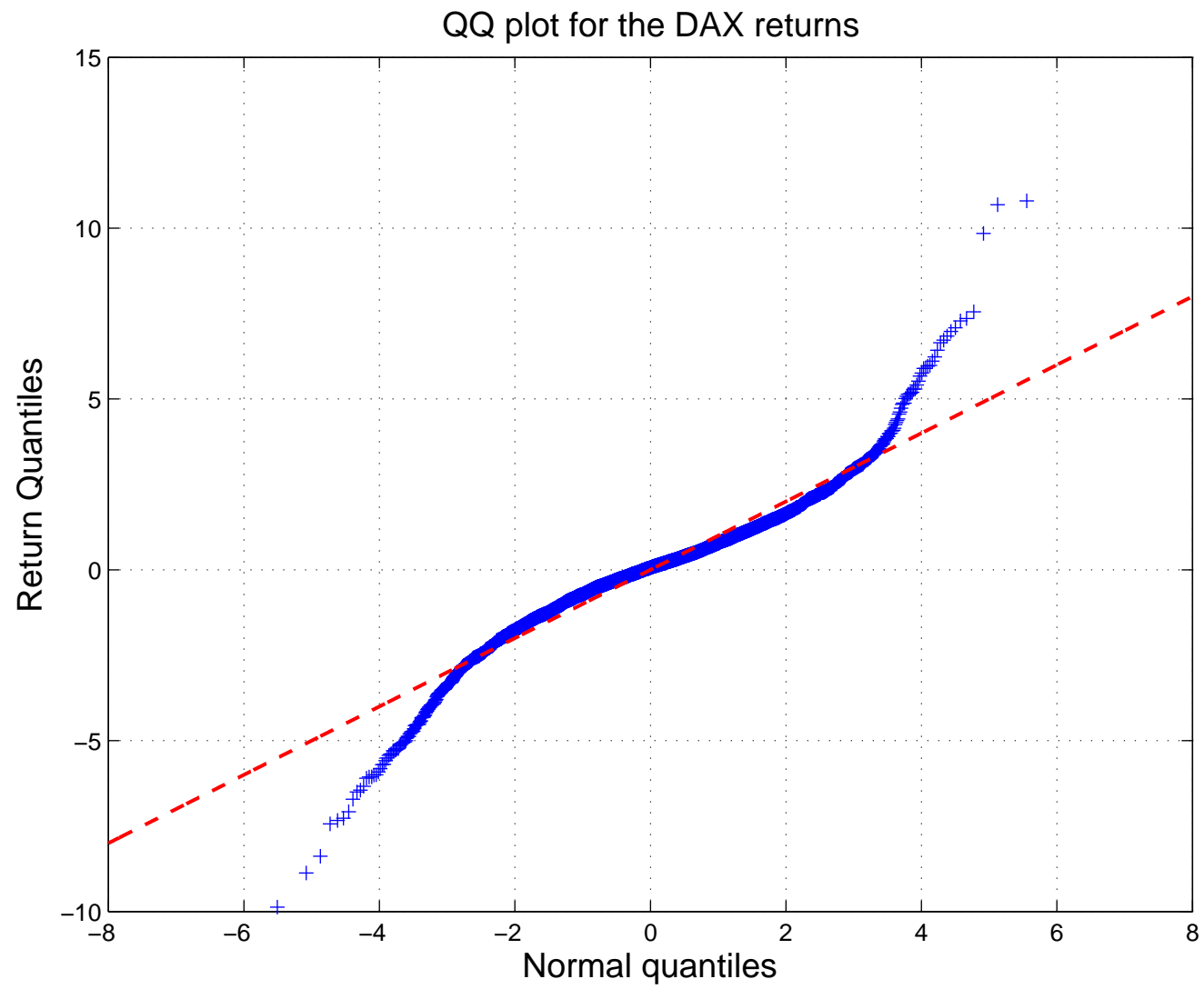
- For the S&P 500 and the DAX, we find

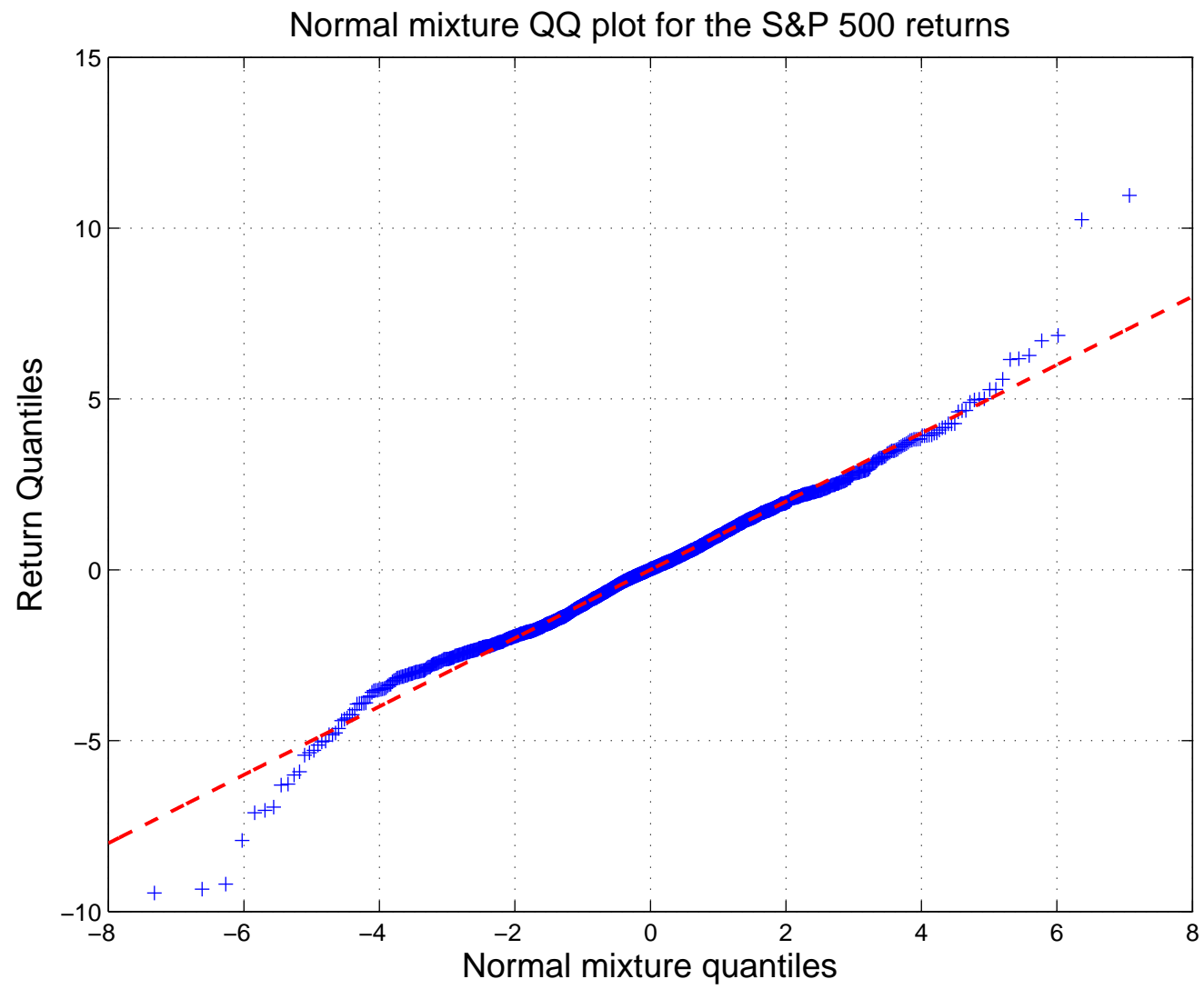
Table 2: Normal mixture parameter estimates

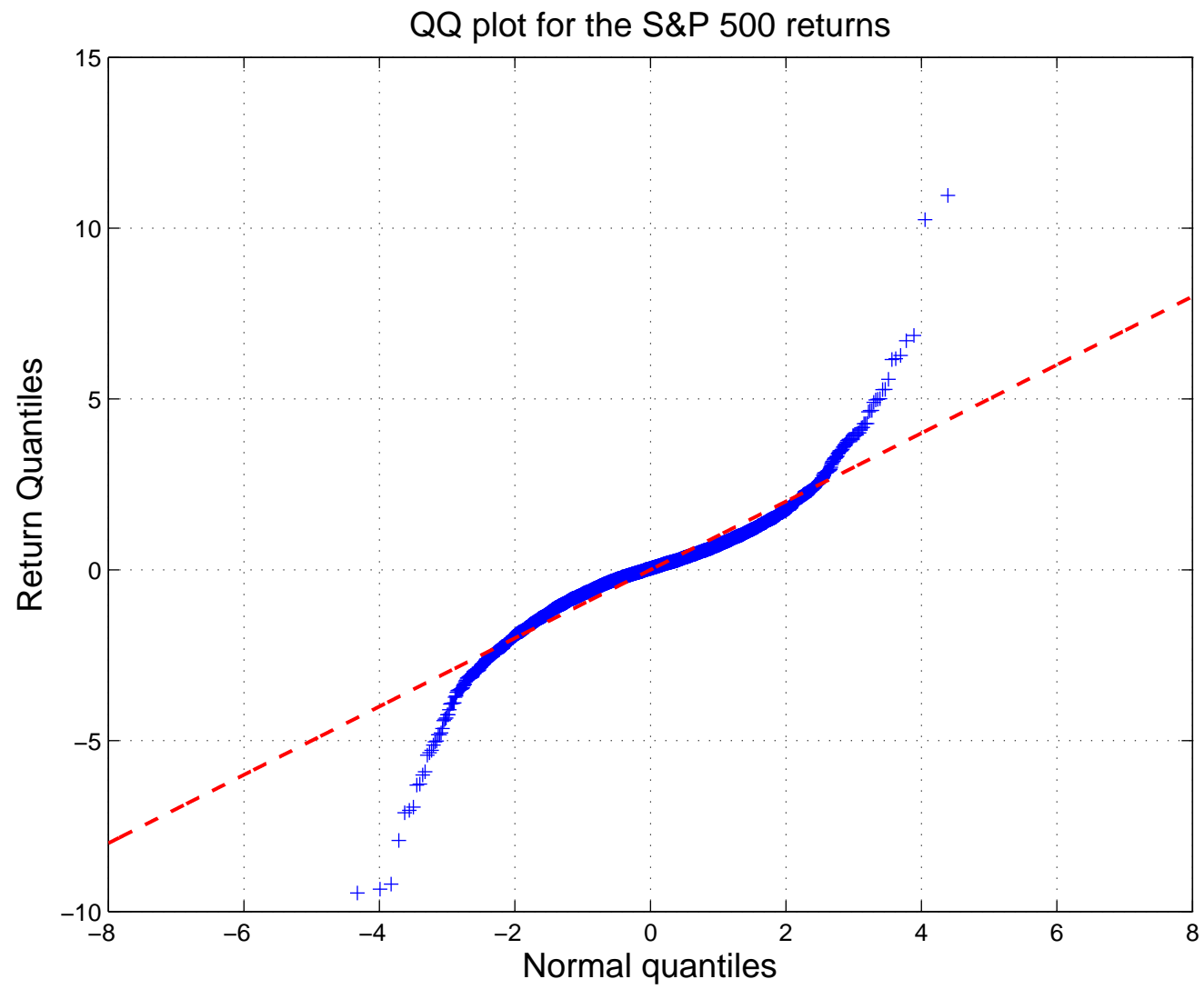
	$\hat{\lambda}_1$	$\hat{\mu}_1$	$\hat{\sigma}_1^2$	$\hat{\lambda}_2$	$\hat{\mu}_2$	$\hat{\sigma}_2^2$
S&P 500	0.801	0.071	0.518	0.199	−0.125	4.771
DAX 30	0.822	0.098	1.025	0.178	−0.285	7.580

- The basic mixture specification (20) can be generalized in various directions to provide a more satisfactory return model.
- For example, the mixing weights can be made time-varying, so that the regimes are persistent or depend on a set of predetermined variables.









Alternative Distributions for Returns: Student's t

- The standard Student's t distribution with mean μ , scale σ and ν degrees of freedom has density

$$f(x) = \frac{\Gamma\left(\frac{\nu+1}{2}\right)}{\Gamma(\nu/2)\sqrt{\pi\nu}\sigma} \left\{ 1 + \frac{(x - \mu)^2}{\nu\sigma^2} \right\}^{-(\nu+1)/2}, \quad \nu > 0. \quad (21)$$

- The variance is $\sigma^2\nu/(\nu - 2)$, and exists only for $\nu > 2$.
- The smaller ν , the fatter the tails, and normality is approached as $\nu \rightarrow \infty$.
- Parameter ν is jointly estimated with the other parameters (e.g., via maximum likelihood).
- (Generalizations that allow for skewness exist.)

Alternative Distributions for Returns

Here $\Gamma(\alpha)$ denotes the gamma function,

$$\Gamma(\alpha) = \int_0^{\infty} x^{\alpha-1} e^{-x} dx, \quad \alpha > 0, \quad (22)$$

with the properties

$$\Gamma(\alpha) = \int_0^{\infty} x^{\alpha-1} e^{-x} dx \quad (23)$$

$$= \underbrace{-x^{\alpha-1} e^{-x} dx \Big|_0^{\infty}}_{=0} + (\alpha - 1) \int_0^{\infty} x^{\alpha-2} e^{-x} dx \quad (24)$$

$$= (\alpha - 1) \Gamma(\alpha - 1), \quad (25)$$

(integration by parts), and thus for $n \in \mathbb{N}$,

$$\Gamma(n) = (n - 1)! = (n - 1)(n - 2) \cdots 1. \quad (26)$$

$$\Gamma(1/2) = \sqrt{\pi}.$$

Alternative Distributions for Returns: Generalized Exponential Distribution (GED)

- This has density

$$f(x) = \frac{2^{-(1/p+1)} p}{\Gamma(1/p) \sigma} \exp \left\{ -\frac{1}{2} \left| \frac{x - \mu}{\sigma} \right|^p \right\}, \quad p > 0, \quad (27)$$

where p measures the thickness of the tails.

- For $p = 2$, this nests the normal, and for $p = 1$ we get the *Laplace* (double exponential) distribution.
- Recall that the normal density with mean μ and variance σ^2 is

$$f(x) = \frac{1}{\sqrt{2\pi}\sigma} \exp \left\{ -\frac{(x - \mu)^2}{2\sigma^2} \right\}. \quad (28)$$

- (Generalizations that allow for skewness exist.)

Concentrating on the Tails

- Often (e.g., when calculating risk measures such as Value-at-Risk) we are not interested in the entire distribution of returns but only in the probability of extreme events.
- It is often found that the tails of return distributions are well described by a power law, i.e., for large x , with F being the distribution function (cdf),

$$1 - F(x) = P(|r| > x) \approx cx^{-\alpha}, \quad (29)$$

where α is the *tail exponent*.

- For the normal distribution, we have, as x becomes large,

$$1 - F(x) \simeq \frac{1}{\sqrt{2\pi}x} \exp \left\{ -\frac{x^2}{2} \right\}, \quad (30)$$

the tails tend to zero faster than exponentially.

- In a log–log plot of the empirical complementary cdf $(1 - F(x))$ against $|r|$ (assuming approximate symmetry), the observations should approximately plot along a straight line (which can be drawn using linear regression,

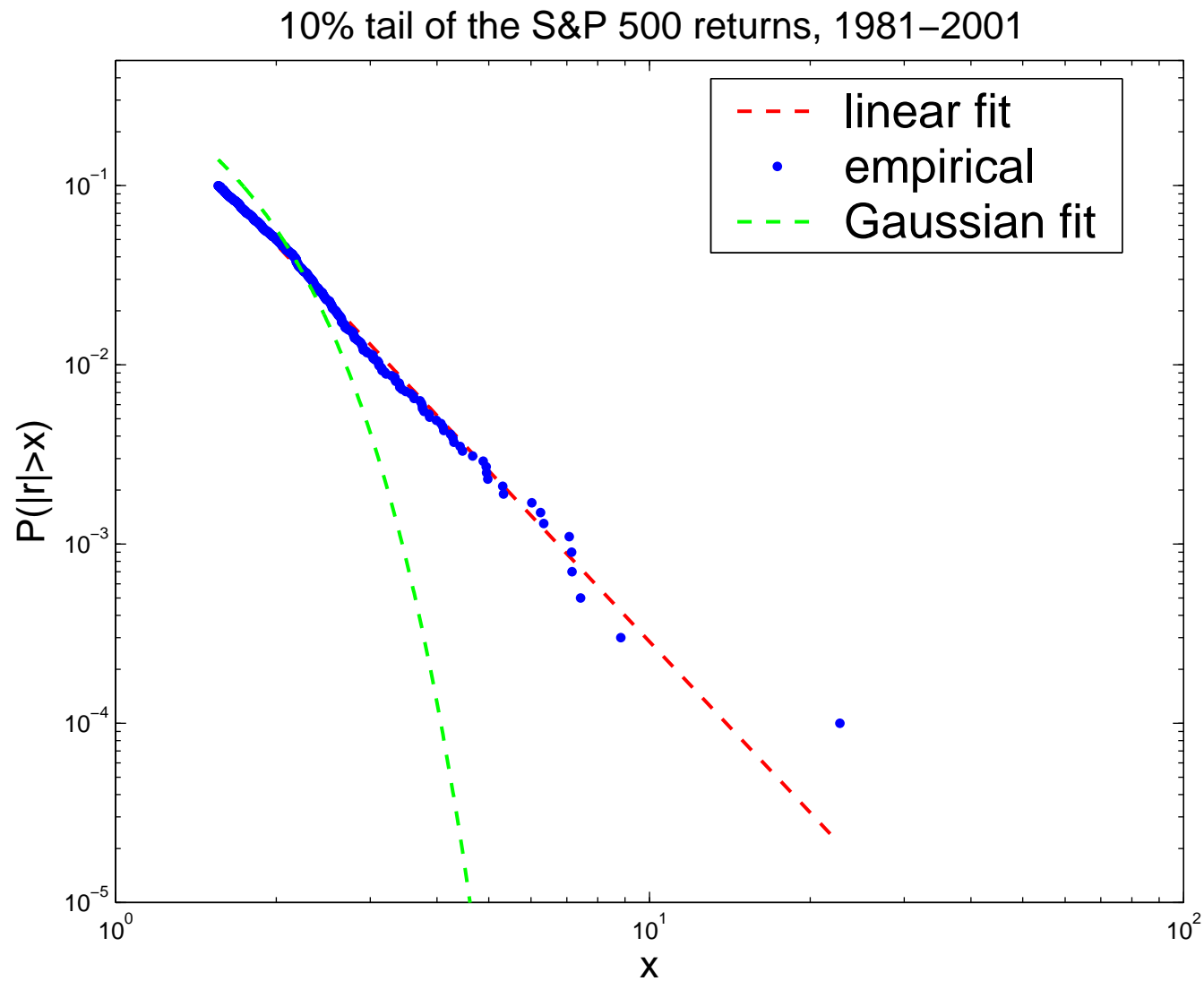
$$\log(1 - F(x)) = \log c - \alpha \log x, \quad (31)$$

the slope parameter is the tail exponent⁴).

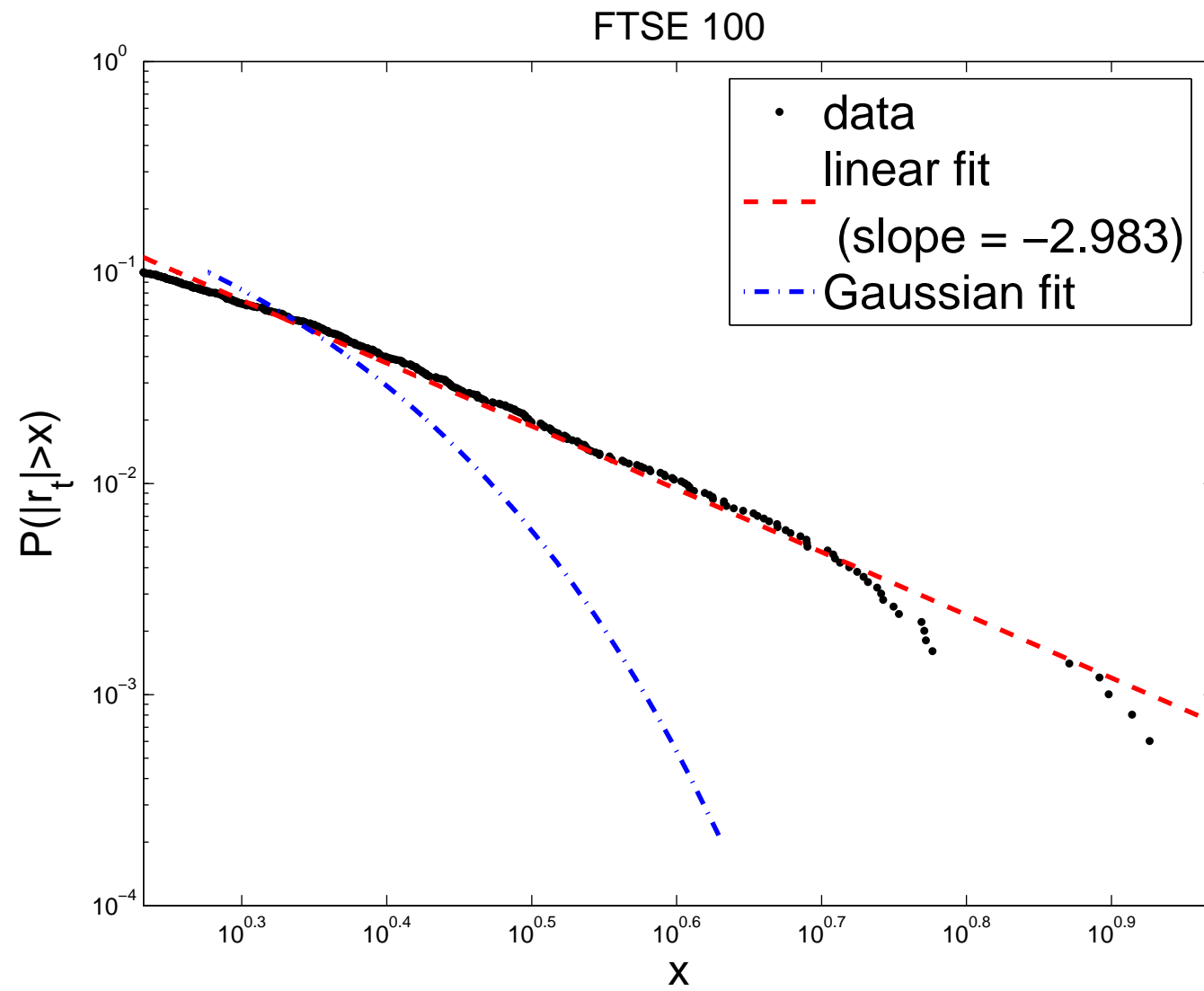
- Several distributions are characterized by power law tails. For example, Student's t has power tails with tail index ν .
- The tail exponent indicates the maximally existing moment, that is

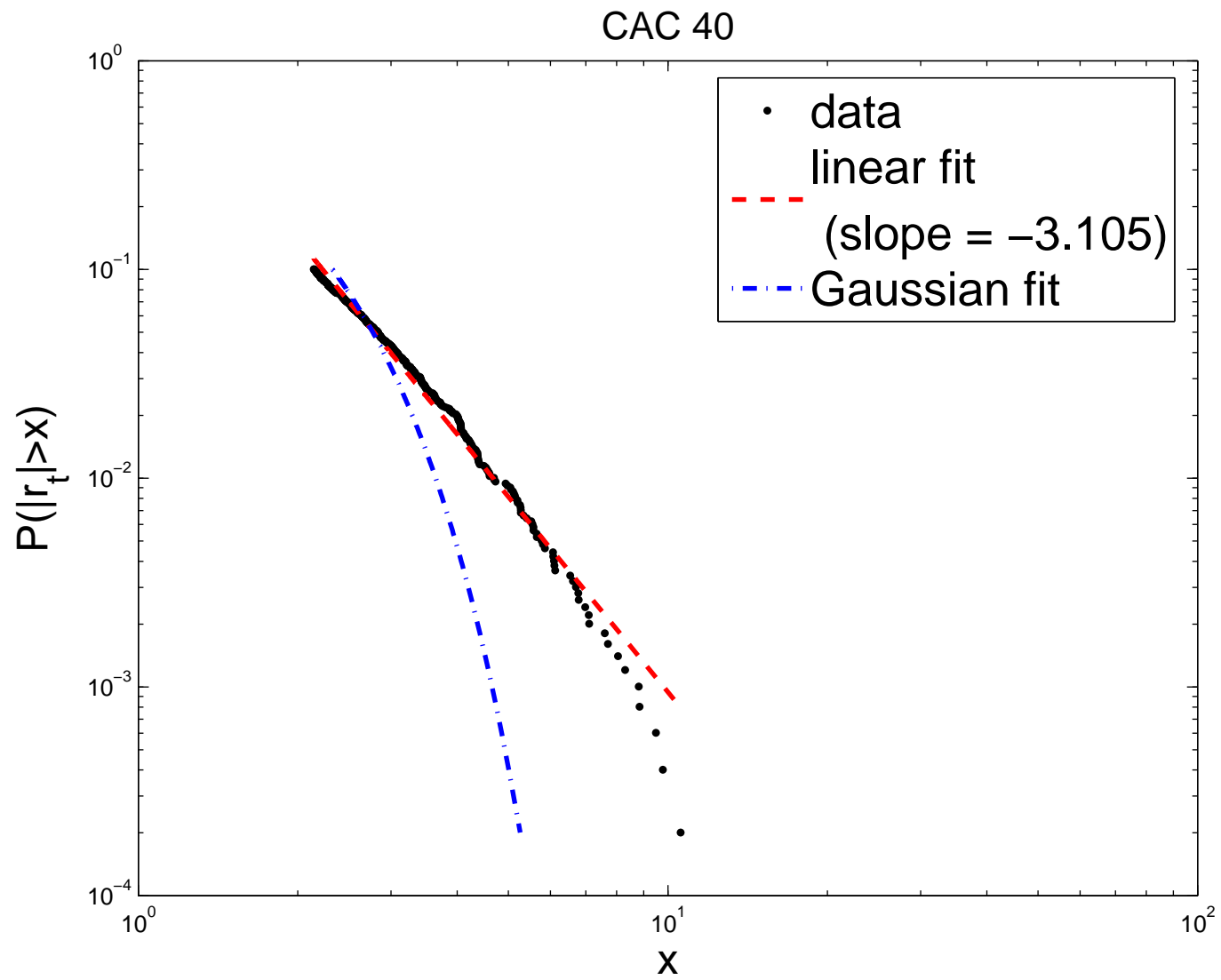
$$\mathbb{E}(|X|^k) = \infty \quad \text{for } k \geq \alpha. \quad (32)$$

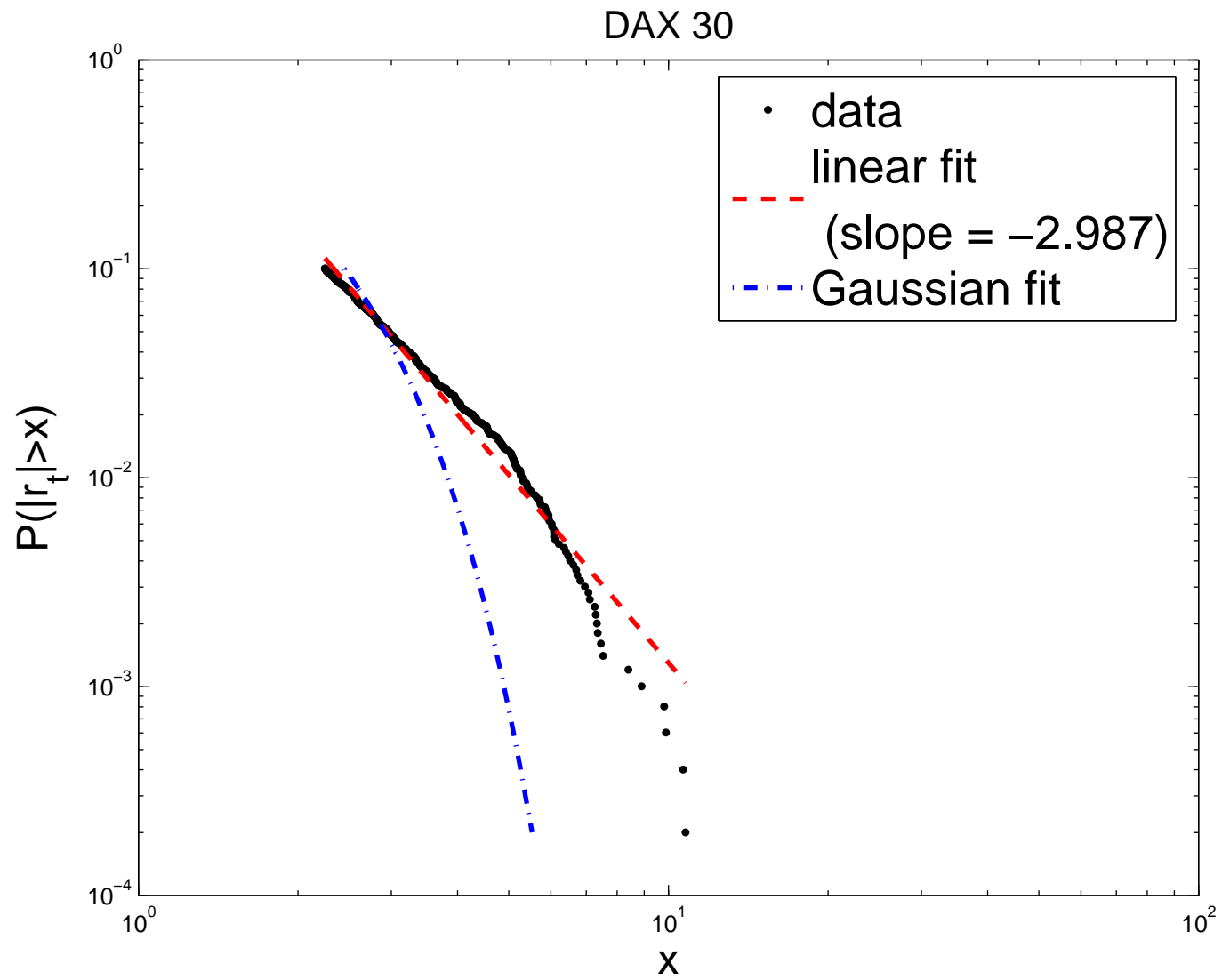
⁴Better estimators of α exist.



Here the (regression-based) estimated tail index is 3.17, which is rather typical for stock returns.







Temporal Properties of Returns

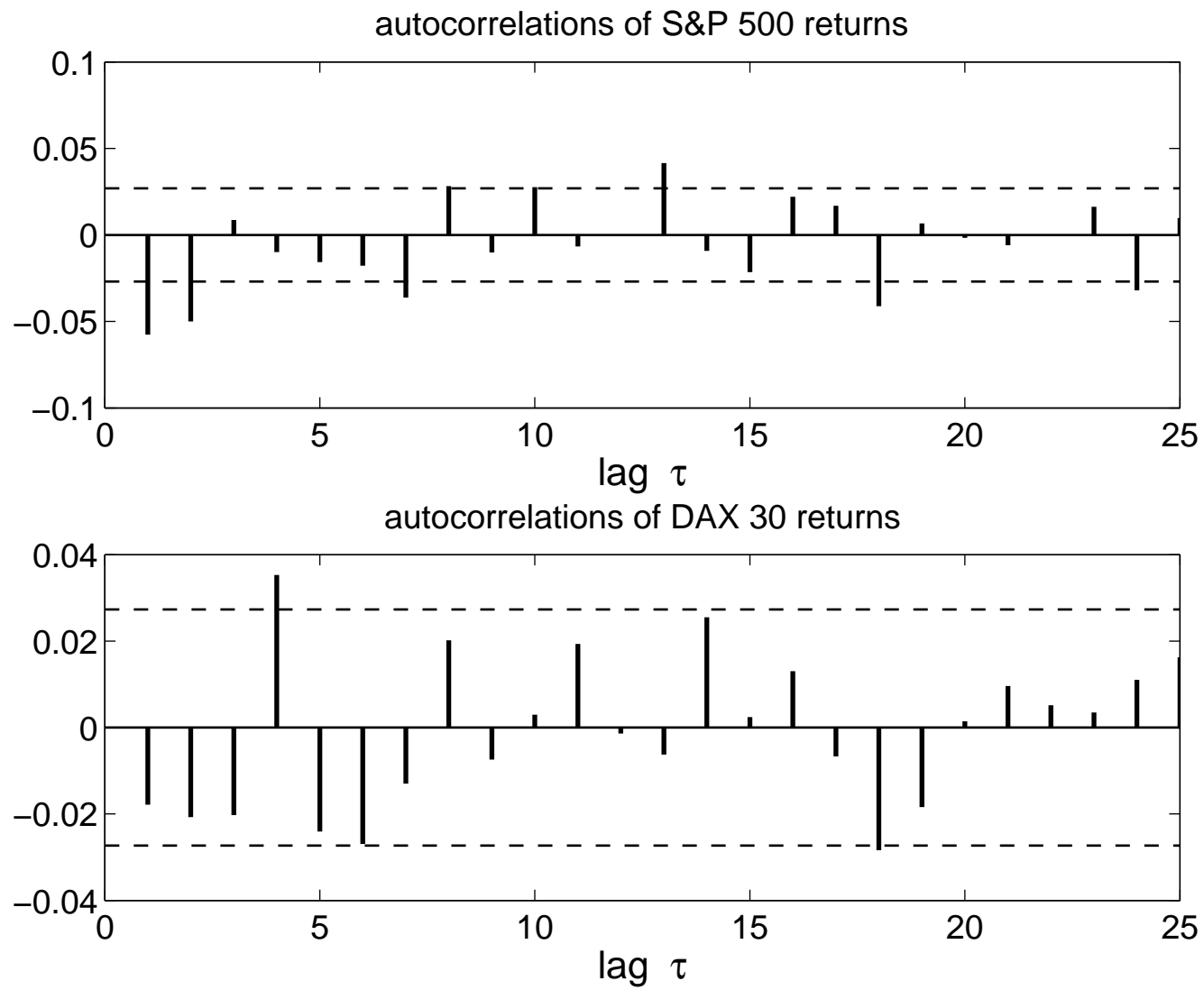
- Consider the sample autocorrelation function at lag τ ,

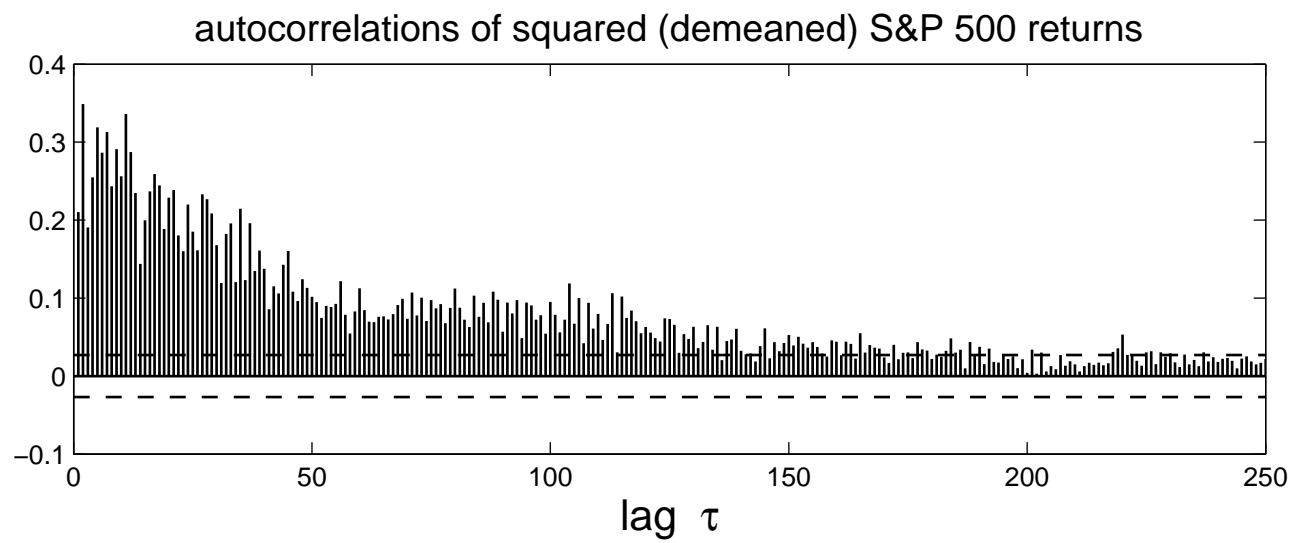
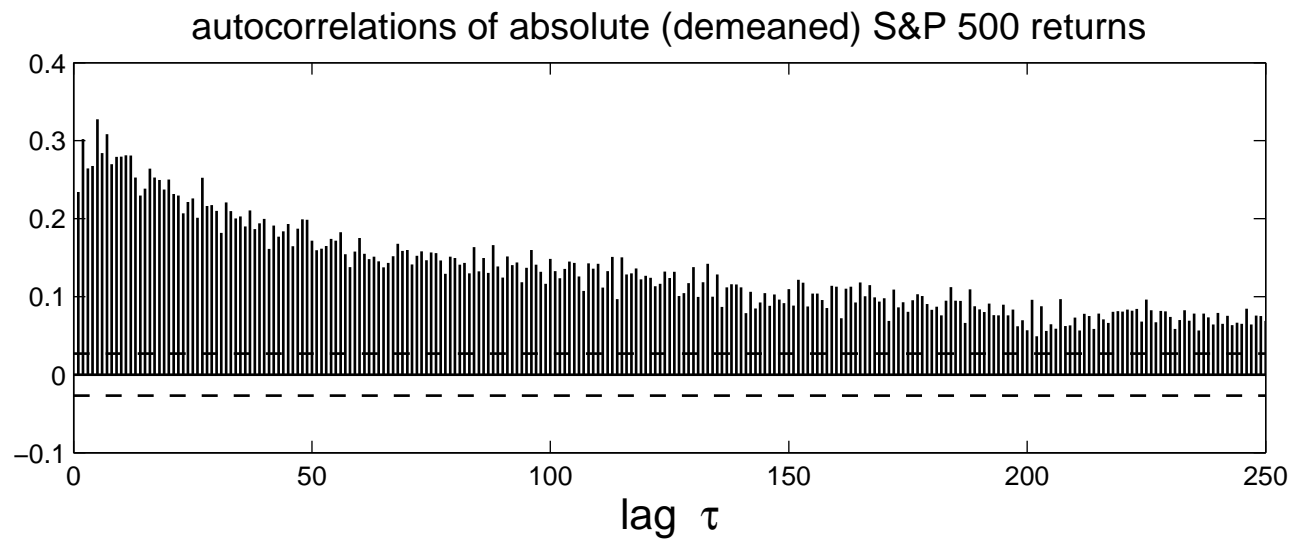
$$\hat{\rho}(\tau) = \frac{\sum_{t=1}^{T-\tau} (r_t - \bar{r})(r_{t+\tau} - \bar{r})}{\sum_{t=1}^T (r_t - \bar{r})^2}, \quad \tau > 0, \quad (33)$$

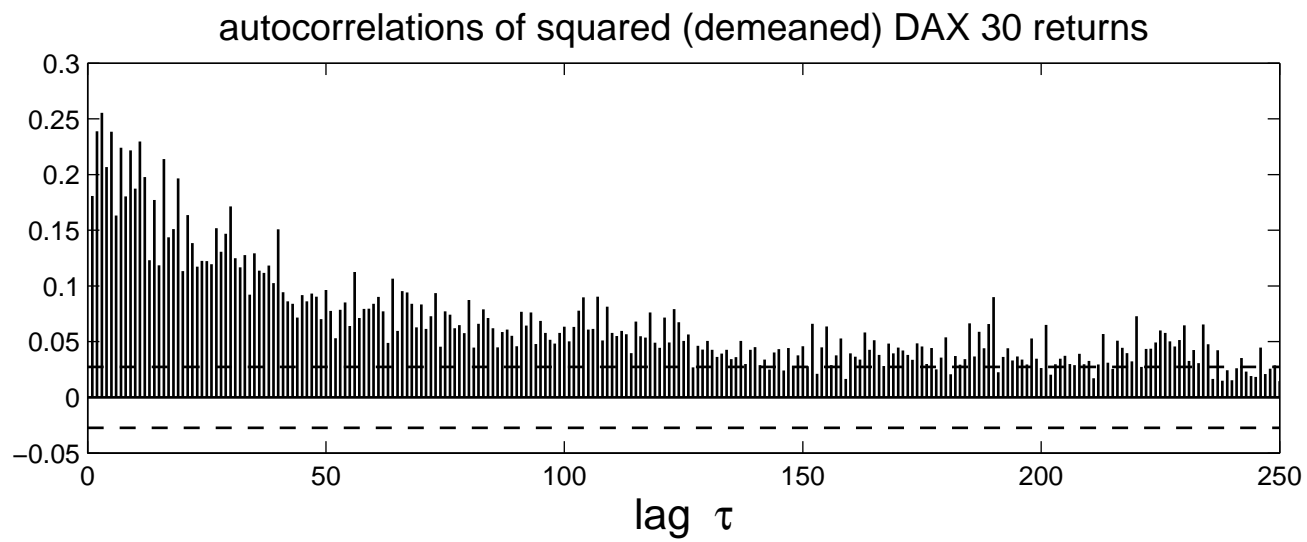
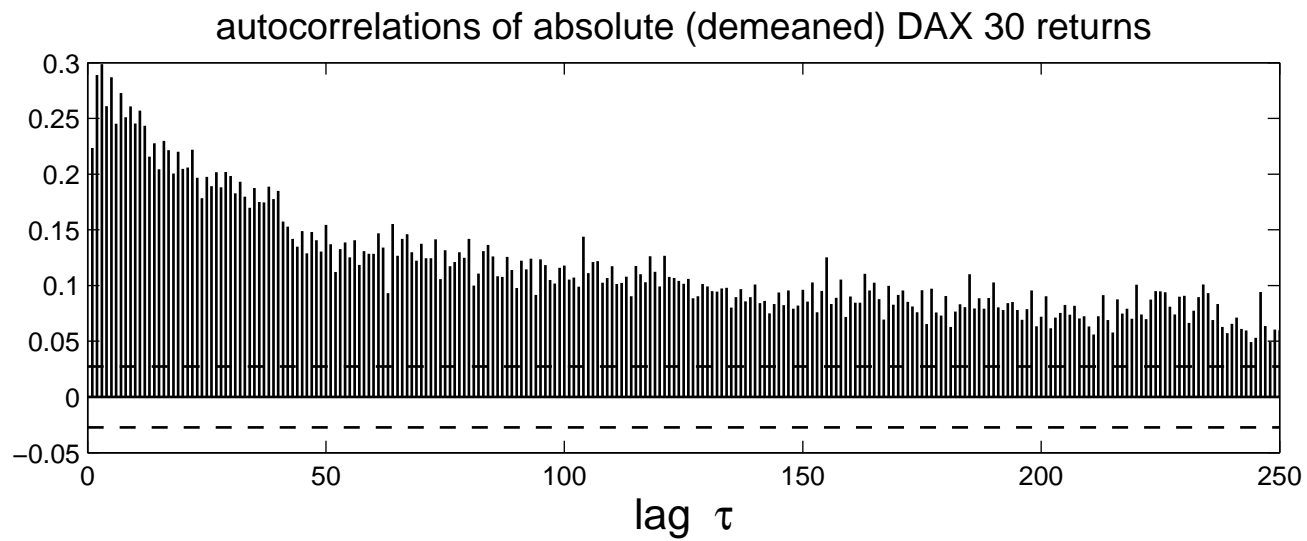
where

$$\bar{r} = \frac{1}{T} \sum_{t=1}^T r_t, \quad (34)$$

and T is the sample size.







Temporal Properties of Returns

- Return series are characterized by *volatility clustering*, that is, “large [price] changes tend to be followed by large changes—of either sign—and small changes tend to be followed by small changes”.⁵
- Thus variance (and thus risk) appears to be persistent and predictable (in contrast to the direction of price changes).
- Several approaches for capturing time-varying volatility have been developed, such as (G)ARCH, stochastic volatility, and regime-switching models.

⁵B. Mandelbrot (1963). The Variation of Certain Speculative Prices. *Journal of Business* 36, 394-419.

Dependence Structure of Returns

- In basic portfolio theory, we are interested in the first two moments of the (portfolio) return distribution, i.e., mean and variance.
- In this framework, correlations between assets are of predominant interest, because the strength of the correlations determines the degree of risk (variance) reduction that can be achieved by efficient portfolio diversification.
- Simple correlation estimates may be misleading, however, due to *asymmetric dependence structures*.
- This refers to the observation that, for example, stock returns are more dependent in bear markets (market downturns) than in bull markets.
- Therefore, diversification might fail when the benefits from diversification are most urgently needed.

Dependence Structure of Returns

- A popular tool to describe this asymmetric dependence structure are the **exceedance correlations** of Longin and Solnik (2001).⁶
- For a given threshold θ , the exceedance correlation between (demeaned) returns r_1 and r_2 is given by

$$\rho(\theta) = \begin{cases} \text{Corr}(x, y | x > \theta, y > \theta) & \text{for } \theta \geq 0 \\ \text{Corr}(x, y | x < \theta, y < \theta) & \text{for } \theta \leq 0 \end{cases} \quad (35)$$

- Let us consider monthly returns of MSCI stock market indices for the US and Germany from January 1970 to June 2008.

⁶Extreme Correlation of International Equity Markets. *Journal of Finance* 56, 649-676.

