Exercise Session - Problem Set 1

Problem 1

For the matrices

$$\mathbf{A} = \begin{pmatrix} 1 & -3 & 5 \\ 2 & -2 & 0 \end{pmatrix} \quad \mathbf{B} = \begin{pmatrix} 7 & 2 \\ 1 & -3 \\ 2 & 4 \end{pmatrix} \quad \mathbf{C} = \begin{pmatrix} 0 & 2 & -1 \\ 4 & -9 & -3 \\ 5 & 2 & 3 \end{pmatrix} \quad \mathbf{D} = \begin{pmatrix} 1 & 7 & 5 \\ 11 & 8 & 2 \\ 2 & 4 & 0 \end{pmatrix},$$

compute i) $\mathbf{AB}$ ii) $\mathbf{BA}$ iii) $\mathbf{A'B'}$ iv) $\mathbf{AC}$ v) $\mathbf{CA}$ vi) $\mathbf{C}$-$\mathbf{D}$ vii) $\mathbf{A}+\mathbf{D}$.

Problem 2 (Based on Wooldridge, Example D.14, p. 818)

a) Use the properties of trace to prove that for any $n \times m$ matrix $tr(\mathbf{A'A}) = tr(\mathbf{AA'})$.

b) Verify that $tr(\mathbf{A'A}) = tr(\mathbf{AA'})$ using matrix $\mathbf{A}$ in Problem 1.

Problem 3

For the matrix

$$\mathbf{A} = \begin{pmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{pmatrix}$$

calculate $|\mathbf{A}|$, $tr(\mathbf{A})$, $\mathbf{A}^{-1}$.

Problem 4

Write the following system of linear equations in matrix notation and solve for the vector $\mathbf{x}$. Under which assumption a nonhomogeneous system of equations will have an unique solution?

$$\begin{array}{rcrcrcr} x_1 & - & x_2 & + & x_3 & = & 2 \\ x_1 & + & x_2 & - & x_3 & = & 0 \\ -x_1 & - & x_2 & - & x_3 & = & 6 \end{array}$$

Problem 5 (Based on Greene, p.12–15 and Example 5, p. 59)

a) Express the sum of the elements in any $n \times 1$ vector $\mathbf{x}$ in matrix terms. Do the same for the case where all elements in $\mathbf{x}$ are equal to the same constant $a \neq 0$.

b) Express the arithmetic mean for any $n \times 1$ vector $\mathbf{x}$.

c) A fundamental matrix in econometrics is the idempotent matrix $\mathbf{M}^0$, which is used to form deviations from sample average. For the $n \times 1$ vector $\mathbf{x}$ derive $\mathbf{M}^0$. What are the properties of idempotent matrices?

d) Express the sum of deviations about the mean for the $n \times 1$ vector $\mathbf{x}$ using $\mathbf{M}^0$. Express also the sum of squared deviations about the mean in matrix terms.

e) Prove that for $K \times 1$ column vectors, $\mathbf{x}_i, i = 1, ..., n$, and some nonzero vector $\mathbf{a}$,

$$\sum_{i=1}^{n} (\mathbf{x}_i - \mathbf{a})(\mathbf{x}_i - \mathbf{a})' = \mathbf{X}'\mathbf{M}^0\mathbf{X} + n(\bar{\mathbf{x}} - \mathbf{a})(\bar{\mathbf{x}} - \mathbf{a})',$$

where the ith row of $\mathbf{X}$ is $\mathbf{x}_i'$ and $\mathbf{M}^0$ is the idempotent matrix defined in c).

[**Hint**: For the solution of a)–d) use a $n \times 1$ vector of ones denoted by $\mathbf{j}_n$]